online

Math 581 Quiz 1 Fall 2021 Name____________________

YOU ARE BEING GRADED FOR WORK

1:23 1) What is a probability space? $(\Omega, \mathcal{F}, P)$ is a probability space if $\Omega$ is a sample space, $\mathcal{F}$ is a $\sigma$-field on $\Omega$, and $P$ is a probability measure on $(\Omega, \mathcal{F})$.

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2) Simplify the following sets. Answers might be $(a,b), [a,b), (a,b], [a,b], [a,a] = \{a\}, (a,a) = \emptyset$.

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i) $\bigcap_{n=1}^{\infty} \left(a, b + \frac{1}{n}\right) = (a, b]$ $= \bigcap_{n \in \mathbb{N}} (a, b + \frac{1}{n})$ $\mathbb{N}$ = set of positive integers

ii) $\bigcap_{n=1}^{\infty} \left[a, b + \frac{1}{n}\right) = [a, b]$

iii) $\bigcap_{n=1}^{\infty} \left[a, a + \frac{1}{n}\right) = [a, a] = \{a\}$

iv) $\bigcup_{n=1}^{\infty} \left(a, b - \frac{1}{n}\right] = (a, b)$

v) $\bigcup_{n=1}^{\infty} \left[a, b - \frac{1}{n}\right] = [a, b)$

Note: $\bigcap_{n \geq m}^{\infty} [a, b - \frac{1}{n}]$ where $m$ is large enough so that for $n \geq m$, $b - \frac{1}{n} \geq a$ would be $[a, b - \frac{1}{m}]$

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3) A DeMorgan's law can be written as $\left[\bigcap_{k=n}^{N} A_k\right]^c = \bigcup_{k=n}^{N} A_k^c$ where $N \geq n$ and $N = \infty$ is allowed.

i) Find $\left[\bigcup_{k=n}^{N} A_k\right]^c$ using the above law and complementation.

$\left[\bigcap_{k=n}^{N} A_k^c\right]^c = \bigcup_{k=n}^{N} A_k$ so $\left[\bigcup_{k=n}^{N} A_k\right]^c = \boxed{\bigcap_{k=n}^{N} A_k^c}$

ii) $limsup_n\ A_n^c = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k^c = \bigcap_{n=1}^{\infty} C_n^c$ where $C_n^c = \bigcup_{k=n}^{\infty} A_k^c$.

Use DeMorgan's law to find $\left[\bigcap_{n=1}^{\infty} C_n^c\right]^c$. $= \bigcup_{n=1}^{\infty} C_n$

iii) Find $C_n$. $= [C_n^c]^c = \left[\bigcup_{k=n}^{\infty} A_k^c\right]^c = \bigcap_{k=n}^{\infty} A_k$

iv) Use the above results to show

$[\ limsup_n\ A_n^c]^c = \left[\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k^c\right]^c = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k = \liminf_n A_n.$

$\left[\limsup_n A_n^c\right]^c = \left[\bigcap_{n=1}^{\infty} C_n^c\right]^c = \bigcup_{n=1}^{\infty} C_n$

$= \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k = \liminf_n A_n$

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