Math 581 Quiz 2 Fall 2021 Name____________________

YOU ARE BEING GRADED FOR WORK

1) Suppose Λ is the index set for $\sigma-$fields on Ω, $\mathcal{F}_\lambda$, that contain a class $\mathcal{A}$ of subsets of Ω. Then Λ is nonempty since the $\sigma-$field of all subsets of Ω contains $\mathcal{A}$. Let the $\sigma-$field generated by $\mathcal{A}$ be

$$\sigma(\mathcal{A}) = \bigcap_{\lambda \in \Lambda} \mathcal{F}_\lambda.$$

Prove that $\sigma(\mathcal{A})$ is a $\sigma-$field.

i) $\Omega \in \mathcal{F}_\lambda \ \forall \lambda \Rightarrow \Omega \in \sigma(\mathcal{A})$

ii) $A \in \sigma(\mathcal{A}) \Rightarrow A \in \mathcal{F}_\lambda \ \forall \lambda \Rightarrow A^c \in \mathcal{F}_\lambda \ \forall \lambda$
$\Rightarrow A^c \in \sigma(\mathcal{A})$

omit iii) $A, B \in \sigma(\mathcal{A}) \Rightarrow A, B \in \mathcal{F}_\lambda \ \forall \lambda \Rightarrow$
$A \cap B \in \mathcal{F}_\lambda \ \forall \lambda \Rightarrow A \cap B \in \sigma(\mathcal{A})$

iv) $A_1, A_2, \ldots \in \sigma(\mathcal{A}) \Rightarrow A_1, A_2 \ldots \in \mathcal{F}_\lambda \ \forall \lambda$
$\Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{F}_\lambda \ \forall \lambda \Rightarrow \bigcup_{i=1}^{\infty} A_i \in \sigma(\mathcal{A})$

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20

2) State the first Borel Cantelli lemma.

2) State the first Borel-Cantelli lemma.

Let $(\Omega, \mathcal{F}, P)$ be fixed and A_n events.

If $\sum_{n=1}^{\infty} P(A_n) < \infty$ (converges), then $P(\limsup_n A_n) = 0$.

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3) Prove $(limsup_n\, A_n)^c = liminf_n\, A_n^c$.

$$\left[\limsup_n A_n\right]^c = \left[\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_n\right]^c = \left[\bigcap_{n=1}^{\infty} \underbrace{\left[\bigcup_{k=n}^{\infty} A_K\right]}_{C_n}\right]^c$$

$$= \bigcup_{n=1}^{\infty} \underbrace{\left[\bigcup_{k=n}^{\infty} A_K\right]^c}_{C_n^c} = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_K^c = \liminf_n A_n^c$$

DeMorgan's Law

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