582 old Quiz?

Math 581 Quiz 7 Fall 2021 Name____________________

1) Let $X_n \sim U(-n, n)$ with cdf $F_n(x) = 0.5 + 0.5x/n$ for $-n < x < n$.
a) Find $F(x)$ such that $F_n(x) \to F(x)$ for all real x.

$F(x) \equiv 0.5$

b) Does $X_n \xrightarrow{D} X$? Explain briefly.

no $F(x)$ is continuous but not a cdf

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2) Let $X_n \sim N(0, \sigma_n^2)$ where $\sigma_n^2 \to \infty$ as $n \to \infty$. Let $\Phi(x)$ be the cdf of a $N(0,1)$ RV. Then the cdf of X_n is $F_n(x) = \Phi(x/\sigma_n)$.
a) Find $F(x)$ such that $F_n(x) \to F(x)$ for all real x.

$\frac{x}{\sigma_n} \to 0 \quad \therefore \quad F(x) = \Phi(0) = 0.5$

b) Does $X_n \xrightarrow{D} X$? Explain briefly.

no $F(x)$ is continuous but not a cdf

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3) Let $Y_1, ..., Y_n$ be iid Poisson (λ) random variables.

4.53a Find the limiting distribution of $\sqrt{n}(\ \overline{Y}_n - c\ )$ for appropriate constant c.

$$\sqrt{n}\,(\underbrace{\overline{Y} - \lambda}_{c}) \xrightarrow{D} N(0, \lambda) \quad \text{by CLT}$$

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4) Suppose that $Y_1, ..., Y_n$ are iid with $E(Y) = (1-\rho)/\rho$ and $V(Y) = (1-\rho)/\rho^2$ where $0 < \rho < 1$.

Find the limiting distribution of

$$\sqrt{n}\ \left(\overline{Y}_n - \frac{1-\rho}{\rho}\right).$$

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4.51a

$$\xrightarrow{D} N\left(0, \frac{1-\rho}{\rho^2}\right) \quad \text{by the CLT}$$

13 5) Suppose $X_1, ..., X_n$ are iid from a distribution with $E(X^k) = 2\theta^k/(k+2)$. Find the limiting distribution of $\sqrt{n}(\ \overline{X}_n - c\ )$ for appropriate constant c. 582 notes 3

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$$E(X) = 2\theta/3 \quad , \quad E[X^2] = \theta^2/2$$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{\theta^2}{2} - \frac{4\theta^2}{9} = \frac{9\theta^2 - 8\theta^2}{18} = \frac{\theta^2}{18}$$

$$\therefore \sqrt{n}\left(\overline{X}_n - \frac{2\theta}{3}\right) \xrightarrow{D} N\left(0, \frac{\theta^2}{18}\right) \quad \text{by the CLT}$$

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