Math 582 HW 1 2022 Due Thursday, Jan. 20.

O2.2 means from the Olive online notes. O2.3a) and O2.12 have solution in Chapter 11 of the online notes. **Problems A)-D).**

A) (Similar to O2.11): Let $X_1, ..., X_n$ be independent and identically distributed (iid) from a Poisson(λ) distribution with $E(X) = \lambda$. Let $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$.

a) Find the limiting distribution of $\sqrt{n}$ ($\overline{X} - \lambda$).

b) Find the limiting distribution of $\sqrt{n}$ [$(\overline{X})^3 - (\lambda)^3$].

B) O2.2: Let $X_1, ..., X_n$ be iid from a normal distribution with unknown mean μ and known variance σ^2. Find the limiting distribution of $\sqrt{n}(\overline{X}^3 - c)$ for an appropriate constant c.

C) O2.3a): Let $X_1, ..., X_n$ be a random sample from a population with pdf

$$f(x) = \begin{cases} \frac{\theta x^{\theta-1}}{3^\theta} & 0 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

The method of moments estimator for θ is $T_n = \dfrac{\overline{X}}{3 - \overline{X}}$. Find the limiting distribution of $\sqrt{n}(T_n - \theta)$ as $n \to \infty$.

D) O2.12. Let $Y_n \sim \chi^2_n$.

a) Find the limiting distribution of $\sqrt{n}\ \left(\dfrac{Y_n}{n} - \ 1 \right)$.

b) Find the limiting distribution of $\sqrt{n}\ \left[\left(\dfrac{Y_n}{n} \right)^3 - \ 1 \right]$.