

This homework has 2 pages, 5 problems.

1) (O2.48): Let Y_1, \dots, Y_n be iid with $E(Y) = \mu$ and $V(Y) = \sigma^2$. Let $g(\mu) = \mu^2$. For $\mu = 0$, find the limiting distribution of $n[(\bar{Y}_n)^2 - 0^2] = n(\bar{Y}_n)^2$ by using the Second Order Delta Method.

2) In earlier courses, you should have used moment generating functions to show that if $Y_n = \sum_{i=1}^n X_i$ where the X_i are iid from a nice distribution, then Y_n has a nice distribution where the nice distributions are the binomial, chi-square, gamma, negative binomial, normal, and Poisson distributions. If $E(X) = \mu$ and $V(X) = \sigma^2$ then by the CLT

$$\sqrt{n}(\bar{X}_n - \mu) \xrightarrow{D} N(0, \sigma^2).$$

Since $\sqrt{n}(\frac{Y_n}{n} - \mu)$ and $\sqrt{n}(\bar{X}_n - \mu)$ have the same distribution,

$$\sqrt{n}\left(\frac{Y_n}{n} - \mu\right) \xrightarrow{D} N(0, \sigma^2)$$

For example, if $Y_n \sim N(n\mu, n\sigma^2)$ then $Y_n \sim \sum_{i=1}^n X_i$ where the X_i are iid $N(\mu, \sigma^2)$. Hence

$$\sqrt{n}\left(\frac{Y_n}{n} - \mu\right) \sim \sqrt{n}(\bar{X}_n - \mu) \xrightarrow{D} N(0, \sigma^2).$$

which should not be surprising since

$$\sqrt{n}\left(\frac{Y_n}{n} - \mu\right) \sim N(0, \sigma^2).$$

Write down the distribution of X_i if

- i) $Y_n \sim \text{BIN}(n, p)$ where BIN stands for binomial.
- ii) $Y_n \sim \chi_n^2$.
- iii) $Y_n \sim G(n\alpha, \beta)$ where G stands for gamma.
- iv) $Y_n \sim \text{NB}(n, p)$ where NB stands for negative binomial.
- v) $Y_n \sim \text{POIS}(n\theta)$ where POIS stands for Poisson.

(Write down the distribution if you know it or can find it. Do not use mgfs unless you can not find the distribution. Exam 1 review will have all of the answers. Also see Olive: Section 1.4.)

3) Suppose that $X_n \sim U(-1/n, 1/n)$.

a) What is the cdf $F_n(x)$ of X_n ?

b) What does $F_n(x)$ converge to? (Hint: consider $x < 0$, $x = 0$ and $x > 0$.)

c) $X_n \xrightarrow{D} X$. What is X ?

Hint: week 2 class notes did a lot of this problem

4) Suppose X_1, \dots, X_n are iid from a distribution with $E(X^k) = 2\theta^k/(k+2)$. Find the limiting distribution of $\sqrt{n}(\bar{X}_n - c)$ for appropriate constant c .

5) Suppose X_n is a discrete random variable with $P(X_n = n) = 1/n$ and $P(X_n = 0) = (n-1)/n$.

a) Show $X_n \xrightarrow{D} X$.

b) Does $E(X_n) \rightarrow E(X)$? Explain briefly.