Math 582 HW 6, 2024 Due Wednesday, Oct. 9.

This homework has 1 page, 5 problems.

1) Show the usual Delta Method is a special case of the Multivariate Delta Method if g is a real function ($d = 1$), T_n is a random variable, θ is a scalar and $\boldsymbol{\Sigma} = \sigma^2$ is a scalar ($k = 1$).
Hint: See Exam 2 review 65).

2) Let $\boldsymbol{X}$ be a $k \times 1$ random vector and $\boldsymbol{X}_n$ be a sequence of $k \times 1$ random vectors and suppose that

$$\boldsymbol{t}^T \boldsymbol{X}_n \xrightarrow{D} \boldsymbol{t}^T \boldsymbol{X}$$

for all $\boldsymbol{t} \in \mathbb{R}^k$. Does $\boldsymbol{X}_n \xrightarrow{D} \boldsymbol{X}$? Explain briefly.

3) Suppose the $k \times 1$ random vector $\boldsymbol{X}_n \xrightarrow{D} N_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Hence the asymptotic distribution of $\boldsymbol{X}_n$ is the multivariate normal MVN $N_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ distribution. Find the d, $\tilde{\boldsymbol{\mu}}$ and $\tilde{\boldsymbol{\Sigma}}$ for the following problem. Let $\boldsymbol{C}^T$ be the transpose of $\boldsymbol{C}$.

Let $\boldsymbol{C}$ be an $m \times k$ matrix, then $\boldsymbol{C}\boldsymbol{X}_n \xrightarrow{D} N_d(\tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{\Sigma}})$.

4) The interquartile range IQR$(n) = \hat{\xi}_{n,0.75} - \hat{\xi}_{n,0.25}$ and is a popular estimator of scale. Use the Exam 2 review 66) and 67) to show that

$$\sqrt{n}\frac{1}{2}(IQR(n) - IQR(Y)) \xrightarrow{D} N(0, \sigma_A^2)$$

where

$$\sigma_A^2 = \frac{1}{64}\left[\frac{3}{[f(\xi_{0.75})]^2} - \frac{2}{f(\xi_{0.75})f(\xi_{0.25})} + \frac{3}{[f(\xi_{0.25})]^2}\right].$$

Hint: $\sigma_A^2 = (-1/2 \;\; 1/2)\boldsymbol{\Sigma}(-1/2 \;\; 1/2)^T$ where $\boldsymbol{\Sigma}$ is obtained from 67) for the 0.25 and 0.75 quantiles.

5) Suppose $\boldsymbol{X}_n$ are $k \times 1$ random vectors with characteristic functions $c_{\boldsymbol{X}_n}(\boldsymbol{t})$. Does $c_{\boldsymbol{X}_n}(\boldsymbol{0}) \to a$ for some constant a? Prove or disprove. Here $\boldsymbol{0}$ is a $k \times 1$ vector of zeroes.