Math 582 Quiz 1 2022 Name___________

pm

1) Let $X_1, ..., X_n$ be independent and identically distributed (iid) from a $N(\mu, \sigma^2)$ distribution. Let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

a) Find the limiting distribution of $\sqrt{n}\,(\,\overline{X} - \mu\,)$.

$\xrightarrow{D} N(0, \sigma^2)$ by CLT

b) Find the limiting distribution of

$$\sqrt{n}\,[\,\frac{1}{\overline{X}} - c\,]$$

for appropriate constant c. You may assume $\mu \neq 0$.

Let $g(x) = \frac{1}{x} = x^{-1}$.

Then $g'(x) = \frac{-1}{x^2}$ and $[g'(\mu)]^2 = \frac{1}{\mu^4}$

$\therefore \sqrt{n}\left(\frac{1}{\overline{x}} - \underbrace{\frac{1}{\mu}}_{c}\right) \xrightarrow{D} N(0, [g'(\mu)]^2\sigma^2)$

$\sim N\left(0, \frac{\sigma^2}{\mu^4}\right)$

$g''(x) = \frac{\partial}{\partial x} -x^{-2} = 2x^{-3}$; $g''(0)$ does not exist

2nd order Delta Method fails

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1M 2) Let $Y_1, ..., Y_n$ be independent and identically distributed (iid) from a distribution with probability density function

$$f(y) = \frac{2y}{\theta^2}$$

for $0 < y \le \theta$ and $f(y) = 0$, otherwise. Then $E(Y) = 2\theta/3$ and $V(Y) = \theta^2/18$.

a) Find the limiting distribution of $\sqrt{n}\ \left(\overline{Y} - c \right)$ for appropriate constant c.

$$\sqrt{n}\left(\overline{Y} - \underbrace{\tfrac{2\theta}{3}}_{c}\right) \xrightarrow{D} N\left(0, \frac{\theta^2}{18}\right)$$

by CLT.

25 b) Find the limiting distribution of $\sqrt{n}\ \left(\log(\ \overline{Y}\ ) - \ d \right)$ for appropriate constant d.

Let $g(\mu) = \log(\mu)$,

$g'(\mu) = \frac{1}{\mu}$ and $\mu = \frac{2\theta}{3}$

So $g'\left(\frac{2\theta}{3}\right) = \frac{3}{2\theta}$ and $(g'(\mu))^2 = \frac{9}{4\theta^2}$

$$\therefore \sqrt{n}\left(\log(\overline{Y}) - \underbrace{\log\left(\tfrac{2\theta}{3}\right)}_{d}\right) \xrightarrow{D}$$

$$N\left(0, (g'(\mu))^2 \frac{\theta^2}{18}\right) \sim$$

$$N\left(0, \frac{9}{4\theta^2}\frac{\theta^2}{18}\right) \sim N\left(0, \frac{1}{8}\right)$$

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