1) Suppose $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$ satisfies the OLS CLT. Let $r_i = Y_i - \hat{Y}_i$ be the *i*th OLS residual for i = 1, ..., n where a constant is in the model. The residual bootstrap draws the residuals with replacement to form the model $\mathbf{Y}^* = \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{r}^*$.

Suppose that for some large fixed m, OLS is fit to find $\hat{\boldsymbol{\beta}}_m$ and the m OLS residuals. Then as $n \to \infty$, the m residuals are drawn with replacement to form $\boldsymbol{Y}^* = \boldsymbol{X} \hat{\boldsymbol{\beta}}_m + r^*$ where $Y_i^* = \boldsymbol{x}_i^T \hat{\boldsymbol{\beta}}_m + r_i^*$ for i = 1, ..., n. This model satisfies the OLS CLT with the r_i^* iid with respect to the bootstrap distribution, $E(r_i^*) = 0$, and $V(r_i^*) = \sigma_m^2 = \frac{1}{m} \sum_{i=1}^m r_i^2 = \frac{m-1}{m} MSE(m)$ where $MSE(m) \xrightarrow{P} \sigma^2 = V(e_i)$ as $m \to \infty$. The bootstrap estimator $\hat{\boldsymbol{\beta}}^*$ is found by fitting OLS to the model $\boldsymbol{Y}^* = \boldsymbol{X} \hat{\boldsymbol{\beta}}_m + \boldsymbol{r}^*$. By the OLS CLT,

$$\sqrt{n}(\hat{\boldsymbol{\beta}}^* - \hat{\boldsymbol{\beta}}_m) \stackrel{D}{\to} \boldsymbol{u}.$$

What is the distribution of u?

2) Show that $\sum_{i=1}^{n} (\boldsymbol{x}_i - \overline{\boldsymbol{x}})(Y_i - \overline{Y}) = \sum_{i=1}^{n} (\boldsymbol{x}_i - \overline{\boldsymbol{x}})Y_i$.

$$\frac{1}{\sqrt{2}}(x-x) = \frac{1}{\sqrt{2}}(x-nx) = \frac{1}{\sqrt{2}}(x-nx) = 0$$

$$= \frac{1}{\sqrt{2}}(x-nx) = \frac{1}{\sqrt{2}}(x-nx) = 0$$

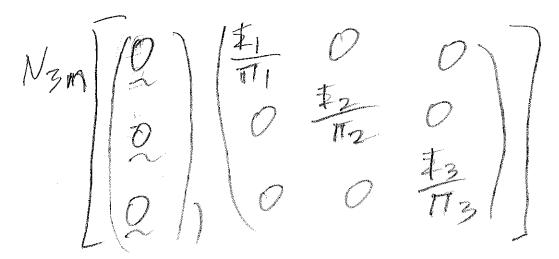
3) Let Σ_i be the nonsingular population covariance matrix of the *i*th treatment group or population. To simplify the large sample theory, assume $n_i = \pi_i n$ where $0 < \pi_i < 1$ and $\sum_{i=1}^{3} \pi_i = 1$. Let T_i be a multivariate location estimator such that

 $\sqrt{n_i}(T_i - \boldsymbol{\mu}_i) \stackrel{D}{\to} N_m(\mathbf{0}, \boldsymbol{\Sigma}_i)$, and $\sqrt{n}(T_i - \boldsymbol{\mu}_i) \stackrel{D}{\to} N_m\left(\mathbf{0}, \frac{\boldsymbol{\Sigma}_i}{\pi_i}\right)$ for i = 1, 2, 3. Assume the T_i are independent.

Then

$$\sqrt{n} \left[egin{array}{l} T_1 - oldsymbol{\mu}_1 \ T_2 - oldsymbol{\mu}_2 \ T_3 - oldsymbol{\mu}_3 \end{array}
ight] \stackrel{D}{
ightarrow} oldsymbol{u}.$$

a) Find the distribution of u.



b) Suggest an estimator $\hat{\pi}_i$ of π_i .

