old quiz 3

Math 595 Quiz 10 Fall 2005 Name___________

1) Suppose that $Y_1, ..., Y_n$ are iid logistic$(\theta, 1)$ with pdf

$$f(y) = \frac{\exp(-(y-\theta))}{[1+\exp(-(y-\theta))]^2}$$

where and y and θ are real.

a) $I_1(\theta) = 1/3$ and the family is regular so the "standard limit theorem" for the MLE $\hat{\theta}_n$ holds. Using this standard theorem, what is the limiting distribution of $\sqrt{n}(\hat{\theta}_n - \theta)$?

$\boxed{\sqrt{n}(\hat{\theta}_n - \theta) \xrightarrow{D} N\left(0, \frac{1}{I_1(\theta)}\right) = N(0, 3)}$

b) Find the limiting distribution of $\sqrt{n}(\overline{Y}_n - \theta)$.

$E Y_i = \theta \qquad V Y_i = \pi^2/3$

So $\boxed{\sqrt{n}(\overline{Y}_n - \theta) \xrightarrow{D} N(0, \pi^2/3)}$ and $\frac{\pi^2}{3} \approx 3.2899$

c) Find the limiting distribution of $\sqrt{n}(MED(n) - \theta)$. $\xrightarrow{D} N\left(0, \frac{1}{4[f(0)]^2}\right)$

$f(\theta) = \frac{1}{(1+1)^2} = \frac{1}{4}$ So $4[f(\theta)]^2 = 4\frac{1}{16} = 4$

and $\frac{1}{4[f(\theta)]^2} = 4$ So $\boxed{\sqrt{n}(MED(n) - \theta) \xrightarrow{D} N(0, 4)}$

d) Consider the estimators $\hat{\theta}_n$, $\overline{Y}_n$ and MED(n). Which is the best estimator and which is the worst? $\hat{\theta}_n$ is best and MED(n) is worst.

smallest asymptotic variance

largest asymptotic variance