

fake home

Math 582

Quiz 4 2022

Name _____

1) Let Y_1, \dots, Y_n be iid uniform $(0, 2\theta)$.

a) Let $T_{1,n} = \bar{Y}$ and find the limiting distribution of $\sqrt{n}(T_{1,n} - \theta)$.

$$E(Y) = \frac{2\theta}{2} = \theta, \quad V(Y) = \frac{(2\theta - 0)^2}{12} = \frac{4\theta^2}{12} = \frac{\theta^2}{3}$$

$$\sqrt{n}(\bar{Y} - \theta) \xrightarrow{D} N\left(0, \frac{\theta^2}{3}\right) \text{ by the CLT}$$

b) Let $T_{2,n} = \text{MED}(n)$ be the sample median and find the limiting distribution of $\sqrt{n}(T_{2,n} - \theta)$. Hint: $\text{MED}(Y) = \theta$.

$$f(y) = \frac{1}{2y} \mathbb{I}(0 < y < 2\theta) \text{ so } f(\text{MED}(Y)) = f(\theta) = \frac{1}{2\theta}$$

$$\sqrt{n}(\text{MED}(n) - \theta) \xrightarrow{D} N\left(0, \frac{1}{4[f(\text{MED}(Y))]^2}\right)$$

$$\sim N\left(0, \frac{1}{4\left(\frac{1}{4\theta^2}\right)}\right) \sim N(0, \theta^2).$$

c) Find $ARE(T_{1,n}, T_{2,n})$. Which estimator is better, asymptotically?

$$= \frac{\sigma_2^2(F)}{\sigma_1^2(F)} = \frac{\theta^2}{\theta^2/3} = 3$$

$\therefore T_{1,n} = \bar{Y}$ is better

pm

2) Let X_n be sequence of random variables with cdfs F_n and mgfs m_n . Let X be a random variable with cdf F and mgf m . Assume that all of the mgfs m_n and m are defined to $|t| \leq d$ for some $d > 0$. Let

$$m_n(t) = \frac{1}{[1 - (\lambda + \frac{1}{n})t]}$$

HW4
problem

for $t < 1/(\lambda + 1/n)$. Show that $m_n(t) \rightarrow m(t)$ by finding $m(t)$.

(Then $X_n \xrightarrow{D} X$ where $X \sim EXP(\lambda)$ with $E(X) = 1/\lambda$ by the continuity theorem for mgfs.)

$$m_n(t) \rightarrow m(t) = \frac{1}{1-\lambda t} \quad \text{for } t < \frac{1}{\lambda} \text{ as } n \rightarrow \infty$$

20

3) Suppose Y_1, \dots, Y_n are iid $POIS(\theta)$. Then $I_1(\theta) = 1/\theta$. Is $\hat{\theta}_n = \bar{Y}_n$ asymptotically efficient? Hint; determine if

efficient for θ

$$\sqrt{n}(\bar{Y}_n - \theta) \xrightarrow{D} N\left(0, \frac{1}{I_1(\theta)}\right)$$

$$E(Y_i) = V(Y_i) = \theta$$

$$\therefore \sqrt{n}(\bar{Y}_n - \theta) \xrightarrow{D} N(0, \theta) \sim N\left(0, \frac{1}{I_1(\theta)}\right)$$

so yes

20