Old Quiz 4

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Math 595 Quiz 5 Fall 2005 Name___________

1) Let $Y_1, ..., Y_n$ be iid exponential $(\lambda)$ so that $E(Y) = \lambda$ and $\text{MED}(Y) = \log(2)\lambda$.

3/3 a) Let $T_{1,n} = \log(2)\overline{Y}$ and find the limiting distribution of $\sqrt{n}(T_{1,n} - \log(2)\lambda)$.

$\sqrt{n}(\overline{Y} - \lambda) \xrightarrow{D} N(0, \lambda^2)$ by the CLT

so $\log(2)\sqrt{n}(\overline{Y} - \lambda) = \sqrt{n}(T_{1n} - \log(2)\lambda) \xrightarrow{D} \log(2) N(0, \lambda^2)$

$= \boxed{N(0, [\log(2)\lambda]^2)}$

by Slutsky's theorem or delta method with $g(\lambda) = \log(2)\lambda$

so $g'(\lambda) = \log(2)$

3/3 b) Let $T_{2,n} = \text{MED}(n)$ be the sample median and find the limiting distribution of $\sqrt{n}(T_{2,n} - \log(2)\lambda)$.

Olive Th 8.5! Lehmann p81

$\sqrt{n}(\text{MED}(n) - \text{MED}(Y)) \xrightarrow{D} N\left(0, \frac{1}{4[f(\text{MED}(Y))]^2}\right)$

$f(y) = \frac{1}{\lambda}\exp\left(\frac{-y}{\lambda}\right)$ so $f(\text{MED}(Y)) = \frac{1}{\lambda}\exp\left(\frac{-\log(2)\lambda}{\lambda}\right) = \frac{1}{\lambda}\exp(-\log 2)$

$= \frac{1}{\lambda}\exp(\log\frac{1}{2}) = \frac{1}{2\lambda}$. So $\frac{1}{4[f(\text{MED}(Y))]^2} = \frac{1}{4\frac{1}{4\lambda^2}} = \lambda^2$

so $\sqrt{n}(T_{2n} - \log(2)\lambda) \xrightarrow{D} \boxed{N(0, \lambda^2)}$

3/4 c) Find $ARE(T_{1,n}, T_{2,n})$. $= \frac{\lambda^2}{[\log(2)\lambda]^2} = \frac{1}{[\log(2)]^2}$

$\approx \frac{1}{.4804} \approx 2.081$

2) Continuity Th

3) asymptotic efficiency

$\log(ab) = \log(a) + \log(b)$