Old Quiz 5

8) Let $W \sim N(\mu_W, \sigma^2_W)$ and let $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. The characteristic function of W is

$$\varphi_W(y) = E(e^{iyW}) = \exp\left(iy\mu_W - \frac{y^2}{2}\sigma^2_w\right).$$

Suppose $W = \boldsymbol{t}^T\boldsymbol{X}$. Then $W \sim N(\mu_W, \sigma^2_W)$. Find μ_W and σ^2_W. Then the characteristic function of $\boldsymbol{X}$ is

$$\varphi_{\boldsymbol{X}}(\boldsymbol{t}) = E(e^{i\boldsymbol{t}^T\boldsymbol{X}}) = \varphi_W(1).$$

Use these results to find $\varphi_{\boldsymbol{X}}(\boldsymbol{t})$.

$\mu_W = E[\underline{t}^T \underline{x}] = \underline{t}^T E(\underline{x}) = \underline{t}^T \underline{\mu}$

$\sigma^2_W = V(\underline{t}^T \underline{x}) = Cov(\underline{t}^T \underline{x}) = \underline{t}^T \Sigma \underline{t}$

$$\boxed{\varphi_{\underline{x}}(\underline{t}) = \exp\left(i\, \underline{t}^T \underline{\mu} - \frac{1}{2} \underline{t}^T \Sigma \underline{t}\right).}$$

$= \exp\left(i \mu_W - \frac{1}{2}\sigma_W^2\right)$

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9) Suppose $\boldsymbol{X}_1, ..., \boldsymbol{X}_n$ are iid $k \times 1$ random vectors where $E(\boldsymbol{X}_i) = \boldsymbol{1} = (1, ..., 1)^T$ and $Cov(\boldsymbol{X}_i) = \boldsymbol{I}_k = diag(1, ..., 1)$, the $k \times k$ identity matrix. Find the limiting distribution of $\sqrt{n}(\overline{\boldsymbol{X}} - \boldsymbol{c})$ for appropriate vector $\boldsymbol{c}$.

$$\boxed{\sqrt{n}(\underline{\overline{X}} - \underline{1}) \xrightarrow{D} N_k(\underline{0}, I_k)}$$

or $\rightarrow$

by MCLT

Slutsky's Th

continuity Th

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