

Old Quiz 6

Math 595

Quiz 9 Fall 2005

Name _____

Similar to HW 6

1) Following the notation of HW9 A), find the limiting distribution of

$$\sqrt{n} \left(\frac{\hat{\xi}_{n,0.25} + \hat{\xi}_{n,0.75}}{2} - \frac{\xi_{0.25} + \xi_{0.75}}{2} \right) = (*)$$

$$\sqrt{n} \left[\begin{pmatrix} \hat{\xi}_{n,0.25} \\ \hat{\xi}_{n,0.75} \end{pmatrix} - \begin{pmatrix} \xi_{0.25} \\ \xi_{0.75} \end{pmatrix} \right] \xrightarrow{D} N_2 \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{3}{16 [f(\xi_{0.25})]^2} & \frac{1}{16 f(\xi_{0.25}) f(\xi_{0.75})} \\ \frac{1}{16 f(\xi_{0.25}) f(\xi_{0.75})} & \frac{3}{16 [f(\xi_{0.75})]^2} \end{pmatrix} \right)$$

\$

So (*) = $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ LHS \xrightarrow{D} $N(0, \sigma_A^2)$

Where $\sigma_A^2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$$= \frac{1}{4} \begin{pmatrix} \frac{3}{16 [f(\xi_{0.25})]^2} + \frac{1}{16 f(\xi_{0.25}) f(\xi_{0.75})} & \frac{1}{16 f(\xi_{0.25}) f(\xi_{0.75})} \\ \frac{1}{16 f(\xi_{0.25}) f(\xi_{0.75})} + \frac{3}{16 [f(\xi_{0.75})]^2} & \frac{1}{16 f(\xi_{0.25}) f(\xi_{0.75})} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{4} \left(\frac{3}{16 [f(\xi_{0.25})]^2} + \frac{2}{16 f(\xi_{0.25}) f(\xi_{0.75})} + \frac{3}{16 [f(\xi_{0.75})]^2} \right)$$

$$= \frac{1}{4} \begin{pmatrix} 1 & 1 \\ \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{4} \left[\sigma_{11} + \sigma_{21} \quad \sigma_{12} + \sigma_{22} \right] \begin{pmatrix} 1 \\ 1 \end{pmatrix} \stackrel{\text{equal}}{=} \frac{1}{4} (\sigma_{11} + \sigma_{12} + \sigma_{21} + \sigma_{22})$$