$D = \begin{pmatrix} \frac{\partial g_1}{\partial \theta_1} & \cdots & \frac{\partial g_1}{\partial \theta_p} \\ \vdots & & \\ \frac{\partial g_p}{\partial \theta_p} & & \frac{\partial g_p}{\partial \theta_p} \end{pmatrix}$

$\sqrt{\theta} = \theta^{\frac{1}{2}} \quad \frac{\partial}{\partial \theta} \theta^{\frac{1}{2}} = \frac{1}{2}\theta^{-\frac{1}{2}} = \frac{1}{2\sqrt{\theta}}$

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pm

1) Suppose

$$\sqrt{n}\left(\begin{pmatrix}\hat\sigma_1^2\\ \vdots \\ \hat\sigma_p^2\end{pmatrix} - \begin{pmatrix}\sigma_1^2\\ \vdots \\ \sigma_p^2\end{pmatrix}\right) \xrightarrow{D} N_p(\mathbf{0}, \mathbf{\Sigma}).$$

$g(\theta) = \begin{pmatrix} \sqrt{\sigma_1^2} \\ \vdots \\ \sqrt{\sigma_p^2} \end{pmatrix}$

Let $\boldsymbol{\theta} = (\sigma_1^2, ..., \sigma_p^2)^T$ and let $\boldsymbol{g}(\boldsymbol{\theta}) = (\sqrt{\sigma_1^2}, ..., \sqrt{\sigma_p^2})^T$. Find $\boldsymbol{D}_{\boldsymbol{g}(\boldsymbol{\theta})}$.

$\theta_i = \sigma_i^2$

$$\boxed{D = \begin{pmatrix} \frac{1}{2\sqrt{\sigma_1^2}} & & \\ & \ddots & \\ & & \frac{1}{2\sqrt{\sigma_p^2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2\sigma_1} & & \\ & \ddots & \\ & & \frac{1}{2\sigma_p} \end{pmatrix}}$$

$$= \begin{pmatrix} \frac{\partial \sqrt{\sigma_1^2}}{\partial \sigma_1^2} & \cdots & \frac{\partial \sqrt{\sigma_1^2}}{\partial \sigma_p^2} \\ \frac{\partial \sqrt{\sigma_p^2}}{\partial \sigma_1^2} & & \frac{\partial \sqrt{\sigma_p^2}}{\partial \sigma_p^2} \end{pmatrix}$$

$\frac{3}{3}$ pm

2) Suppose

$$\sqrt{n}\left(\begin{pmatrix}\hat\sigma_1\\ \vdots \\ \hat\sigma_p\end{pmatrix} - \begin{pmatrix}\sigma_1\\ \vdots \\ \sigma_p\end{pmatrix}\right) \xrightarrow{D} N_p(\mathbf{0}, \mathbf{\Sigma}).$$

$g(\theta) = \begin{pmatrix} \sigma_1^2 \\ \vdots \\ \sigma_p^2 \end{pmatrix}$

Let $\boldsymbol{\theta} = (\sigma_1, ..., \sigma_p)^T$ and let $\boldsymbol{g}(\boldsymbol{\theta}) = ((\sigma_1)^2, ..., (\sigma_p)^2)^T$. Find $\boldsymbol{D}_{\boldsymbol{g}(\boldsymbol{\theta})}$.

$\theta_i = \sigma_i$

$$\boxed{D = \begin{pmatrix} 2\sigma_1 & & \\ & \ddots & \\ & & 2\sigma_p \end{pmatrix}} = \begin{pmatrix} \frac{\partial (\sigma_1)^2}{\partial \sigma_1} & \frac{\partial (\sigma_1)^2}{\partial \sigma_2} & \cdots & \frac{\partial (\sigma_1)^2}{\partial \sigma_p} \\ \vdots & & & \\ \frac{\partial (\sigma_p)^2}{\partial \sigma_1} & & \cdots & \frac{\partial (\sigma_p)^2}{\partial \sigma_p} \end{pmatrix}$$

$g(\theta) = \theta^2 \quad g'(\theta) = 2\theta$

$\frac{}{33}$

3.3 ignores TS structure
3.2 Cheb
3.4 short
3.6 modeled short

Table 1: 1 step ahead 95% PI after model selection, MA(2) true

n	dist	PI 3.4	PI 3.6	PI 3.3	PI 3.2
100	N	0.9548	0.9562	0.9412	0.9436
100		4.3870	4.2758	3.8770	3.9250
100	t5	0.9486	0.9502	0.9402	0.9434
100		5.6495	5.5786	4.9980	5.0597
100	U	0.9744	0.9828	0.9904	0.9916
100		2.3104	2.1587	2.2479	2.2758
100	sExp	0.9556	0.9620	0.9482	0.9492
100		3.8550	3.6475	3.8582	3.9167

3) The above table shows simulation results for time series after model selection. The PIs are for Y_{n+1} given training data $Y_1, ..., Y_n$ with $n = 100$. There are 4 PIs 3.4, 3.6, 3.3, and 3.2. There are 4 distributions for the errors N for $N(0, \sigma^2)$, t5 for t_5, U for $U(-1, 1)$, and sEXP for $EXP(1) - 1$. For each distribution there are two lines. The first line gives the coverages and the second line gives the average PI length. Hence for the normal distribution, PI 3.4 had simulated coverage 0.9548 and ave. length = 4.3870 while for the shifted exponential distribution, PI 3.2 had coverage 0.9492 and average length 3.9167. There were 5000 runs, so say the PI is best if its coverage ≥ 0.94 with shortest average length. Which PI is best for the following distributions?

a) N

3.3

b) t5 3.3

c) U 3.6

d) sExp 3.6

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