Math 582 Quiz 9 2022 Name___________

For this quiz, use the following results. A) Suppose $\boldsymbol{X} \sim N_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then
i) $\boldsymbol{AX} \sim N_q(\boldsymbol{A\mu}, \boldsymbol{A\Sigma A}^T)$.
ii) $\boldsymbol{a} + b\boldsymbol{X} \sim N_k(\boldsymbol{a} + b\boldsymbol{\mu}, b^2\boldsymbol{\Sigma})$.
iii) $\boldsymbol{AX} + \boldsymbol{d} \sim N_q(\boldsymbol{A\mu} + \boldsymbol{d}, \boldsymbol{A\Sigma A}^T)$.
(Find the mean and covariance matrix of the left hand side and plug in those values for the right hand side. **Be careful with the dimension** k or q.)

B) Suppose $\boldsymbol{X}_n \xrightarrow{D} N_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Then
i) $\boldsymbol{AX}_n \xrightarrow{D} N_q(\boldsymbol{A\mu}, \boldsymbol{A\Sigma A}^T)$.
ii) $\boldsymbol{a} + b\boldsymbol{X}_n \xrightarrow{D} N_k(\boldsymbol{a} + b\boldsymbol{\mu}, b^2\boldsymbol{\Sigma})$.
iii) $\boldsymbol{AX}_n + \boldsymbol{d} \xrightarrow{D} N_q(\boldsymbol{A\mu} + \boldsymbol{d}, \boldsymbol{A\Sigma A}^T)$.
(The behavior of convergence in distribution to a MVN distribution is much like the behavior of the MVN distributions in A).)

1) By the OLS CLT, $\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \xrightarrow{D} N_p(\boldsymbol{0}, \sigma^2\ \boldsymbol{W})$. Hence the limiting distribution of of $\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$ is the $N_p(\boldsymbol{0}, \sigma^2\ \boldsymbol{W})$ distribution. Let $\boldsymbol{A}$ be a constant $r \times p$ matrix. Find the limiting distribution of $\boldsymbol{A}\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$.

$\xrightarrow{D} N_r(\underset{\sim}{0}, \sigma^2 A W A^T)$

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2) Suppose $\boldsymbol{Z}_n \xrightarrow{D} N_k(\boldsymbol{\mu}, \boldsymbol{I})$. Let $\boldsymbol{A}$ be a constant $r \times k$ matrix. Find the limiting distribution of $\boldsymbol{A}(\boldsymbol{Z}_n - \boldsymbol{\mu})$.

pm

$\xrightarrow{D} N_r(\underset{\sim}{0}, A A^T)$.

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3) For the parametric bootstrap, $\boldsymbol{Y}^* = \boldsymbol{X}\hat{\boldsymbol{\beta}} + \boldsymbol{e}^*$ where $\boldsymbol{e}^* = (e_1^*, ..., e_m^*)^T$ and the e_i^* are iid $N(0, \sigma_n^2)$ random variables. Assume this model satisfies the OLS CLT as $m \to \infty$ where the OLS estimator is $\hat{\boldsymbol{\beta}}^*$ and $\boldsymbol{\beta}$ is replaced by $\hat{\boldsymbol{\beta}}$. Note that σ^2 is replaced by $\hat{\sigma}_n^2$. Assume $\hat{\boldsymbol{\beta}}$ is a $p \times 1$ vector and $\boldsymbol{Y}^*$ is an $m \times 1$ vector. a) Then use the OLS CLT to find the limiting distribution of $\sqrt{m}(\hat{\boldsymbol{\beta}}^* - \hat{\boldsymbol{\beta}})$ as $m \to \infty$.

σ_n^2

or $\hat{\sigma}_n^2$

$$\xrightarrow{D} N_p(\underline{0}, \underbrace{\sigma_n^2 \, W}_{})$$

$$\Sigma_n \to \sigma^2 W$$

b) In a) you should get $\sqrt{m}(\hat{\boldsymbol{\beta}}^* - \hat{\boldsymbol{\beta}}) \xrightarrow{D} N_p(\boldsymbol{0}, \boldsymbol{\Sigma}_n)$ where $\boldsymbol{\Sigma}_n \xrightarrow{P} \boldsymbol{\Sigma}$. The bootstrap proof technique says that suppose $\sqrt{n}(T_n - \boldsymbol{\theta}) \xrightarrow{D} N_g(\boldsymbol{0}, \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma}_n \xrightarrow{P} \boldsymbol{\Sigma}$ as $n \to \infty$, and for fixed n, $\sqrt{m}(T_{n,m}^* - T_n) \xrightarrow{D} N_g(\boldsymbol{0}, \boldsymbol{\Sigma}_n)$ as $m \to \infty$. Then $\sqrt{n}(T_n^* - T_n) \xrightarrow{D} N_g(\boldsymbol{0}, \boldsymbol{\Sigma})$ as $n \to \infty$. Find the limiting distribution $\sqrt{n}(\hat{\boldsymbol{\beta}}^* - \hat{\boldsymbol{\beta}})$ as $n \to \infty$ where you may plug in $\boldsymbol{\Sigma}$ in to the result (you do not need to compute $\boldsymbol{\Sigma}$).

$$\sqrt{n}(\hat{\beta}^* - \hat{\beta}) \xrightarrow{D} N_p(0, \Sigma)$$

$$\sim N_p(\underline{0}, \sigma^2 W)$$

either

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