

sample percentile of the D_k^2 .

LS 72

15}	pred reg	cont reg	
7}	Percentile PI	percentile method CI	10}
9}	Shorth PI	Shorth CI	11}
17a)	MVN pred reg	Standard bootstrap Cont reg (almost 100% applied to T_n^* if T_n is replaced by T_n^*)	12}
17b)	non parametric pred reg	prediction region method cont reg	13}

olive
ch 5

olive
ch 4

The confidence regions 10}, 11}, 13} are obtained by applying the above (ch 4) prediction regions to $T_1^*, \dots, T_B^*$.
(Hence these cont regions are also large sample prediction regions for T_{B+1}^* .)

16} The large sample $100(1-\delta)\%$ BR confidence region is $\{ \underline{w} : n(\underline{w} - T_n)^T C_n^{-1} (\underline{w} - T_n) \leq D_{(BT)}^2 \}$
for $\underline{\theta}$

$$= \left\{ \underline{w} : D_{\underline{w}}^2 \left(T_n, \frac{c_n}{n} \right) \leq D_{(U_{BT})}^2 \right\} \quad 72.5$$

where $D_{(U_{BT})}^2$ is the $100\alpha_B$ th sample

percentile of $D_i^2 = n (T_i^* - T_n) C_n^{-1} (T_i^* - T_n)$

17) The large sample $100(1-\alpha_B)\%$ PR confidence region for $\underline{\theta}$ is

$$\left\{ \underline{w} : n (\underline{w} - \bar{T}^*)^T C_n^{-1} (\underline{w} - \bar{T}^*) \leq D_{(U_B)}^2 \right\}$$

$$= \left\{ \underline{w} : D_{\underline{w}}^2 \left(\bar{T}^*, \frac{c_n}{n} \right) \leq D_{(U_B)}^2 \right\}$$

where $D_{(U_B)}^2$ is the $100\alpha_B$ th sample percentile

of $D_i^2 = n (T_i^* - \bar{T}^*)^T C_n^{-1} (T_i^* - \bar{T}^*)$.

18) Suppose $\sqrt{n} (T_n - \underline{\theta}) \xrightarrow{D} \underline{U}$ and

$\sqrt{n} (T_i^* - T_n) \xrightarrow{D} \underline{U}$ with $E[\underline{U}] = \underline{0}$

and $\text{cov}(\underline{U}) = \Sigma_{\underline{U}} > 0$.

Then $\sqrt{n} (\bar{T}^* - \underline{\theta}) \xrightarrow{D} \underline{U}$, $\sqrt{n} (T_i^* - \bar{T}^*) \xrightarrow{D} \underline{U}$

and $\sqrt{n} (\bar{T}^* - T_n) \xrightarrow{D} \underline{0}$.

proof: wrt the bootstrap sample, LS 73

T_n is a constant and the

$\sqrt{n}(\bar{T}_i^* - T_n)$ are iid for $i=1, \dots, B$. For B

Then
$$\begin{bmatrix} \sqrt{n}(\bar{T}_1^* - T_n) \\ \vdots \\ \sqrt{n}(\bar{T}_B^* - T_n) \end{bmatrix} \xrightarrow{D} \begin{pmatrix} \underline{v}_1 \\ \vdots \\ \underline{v}_B \end{pmatrix} \quad \text{where}$$

the $\underline{v}_i$ are iid with the same dist as $\underline{v}$.

For fixed B , the ave of $\sqrt{n}(\bar{T}_i^* - T_n)$ is $\bar{v} = \frac{1}{B} \sum_{i=1}^B \underline{v}_i$. point mass at 0
 $\rightarrow N(0, 0) \sim 0$ as $B \rightarrow \infty$

$$\sqrt{n}(\bar{T}^* - T_n) \xrightarrow{D} \frac{1}{B} \sum_{i=1}^B \underline{v}_i \sim \text{ANG} \left(0, \frac{\Sigma_v}{B} \right)$$

Let $B \rightarrow \infty$, then $\sqrt{n}(\bar{T}^* - T_n) \xrightarrow{P} 0$.

Note if $\underline{v}_i \sim N_g(0, \Sigma_v)$,

then $\frac{1}{B} \sum_{i=1}^B \underline{v}_i \sim N_g(0, \frac{\Sigma_v}{B})$, and

$$\sqrt{n} \sqrt{B} (\bar{T}^* - T_n) \xrightarrow{D} N_g(0, \Sigma_v).$$

$$\text{So } (\bar{T}^* - T_n) = O_p[(nB)^{-1/2}]$$

$(\sqrt{B} \frac{1}{B} \sum_{i=1}^B \underline{v}_i \xrightarrow{D} N_g(0, \Sigma_v))$ by MCLT as $B \rightarrow \infty$

See HW 8 to complete the proof.

73.5

19] If $\sqrt{n}(\bar{T}_n - \theta) \xrightarrow{D} U$ and $\sqrt{n}(\bar{T}_i^* - \bar{T}_n) \xrightarrow{D} U$ where U has a unimodal pdf symmetric about 0, then the shorth CI, usual percentile CI, and some of the CIs from the conf regions are asymptotically equivalent since they use the central portion of the bootstrap sample, asymptotically.

20] Suppose $\sqrt{n}(\bar{T}_n - \theta) \xrightarrow{D} U$, $\sqrt{n}(\bar{T}_i^* - \bar{T}_n) \xrightarrow{D} U$, $\sqrt{n}(\bar{T}^* - \theta) \xrightarrow{D} U$ and $\sqrt{n}(\bar{T}_i^* - \bar{T}^*) \xrightarrow{D} U$.

Suppose $C_n^{-1} \xrightarrow{P} C^{-1}$. Then

$$D_1^2 = D_{\bar{T}_i^*}^2(\bar{T}^*, \frac{C_n}{n}) = \sqrt{n}(\bar{T}_i^* - \bar{T}^*)^T C_n^{-1} \sqrt{n}(\bar{T}_i^* - \bar{T}^*)$$

$$D_2^2 = D_{\theta}^2(\bar{T}_n, \frac{C_n}{n}) = \sqrt{n}(\bar{T}_n - \theta)^T C_n^{-1} \sqrt{n}(\bar{T}_n - \theta)$$

$$D_3^2 = D_{\theta}^2(\bar{T}^*, \frac{C_n}{n}) = \sqrt{n}(\bar{T}^* - \theta)^T C_n^{-1} \sqrt{n}(\bar{T}^* - \theta)$$

$$D_4^2 = D_{\bar{T}_i^*}^2(\bar{T}_n, \frac{C_n}{n}) = \sqrt{n}(\bar{T}_i^* - \bar{T}_n)^T C_n^{-1} \sqrt{n}(\bar{T}_i^* - \bar{T}_n)$$

and $D_j^2 \xrightarrow{D} D^2 = U^T C^{-1} U$. Hence

16] and 17] are large sample conf reg's.

If C_n^{-1} is "not too ill conditioned" LS 74

then $D_j^2 \approx \underline{U}^T C_n^{-1} \underline{U}$ and

16] and 17] will have coverage near 15

21] $C_n = I_g = C_n^{-1}$ is an interesting choice.

22] The Bootstrap Proof Technique: ^(BPT) Suppose

$\sqrt{n} (T_n - \theta) \xrightarrow{D} N_g(\underline{0}, \Sigma)$ and $\Sigma_n \xrightarrow{P} \Sigma$ as $n \rightarrow \infty$,

and for fixed m , $\sqrt{m} (T_{n,m}^* - T_n) \xrightarrow{D} N_g(\underline{0}, \Sigma_n)$

as $m \rightarrow \infty$.

a) $\sqrt{m} (T_{n,m}^* - T_n) \xrightarrow{D} N_g(\underline{0}, \Sigma)$ as $m, n \rightarrow \infty$.

b) $\sqrt{n} (T_n^* - T_n) \xrightarrow{D} N_g(\underline{0}, \Sigma)$ as $n \rightarrow \infty$

where $T_n^* = T_{n,n}^*$ has $m=n$.

Proof} By the three assumptions,

$\underline{U}_n = \sqrt{n} (T_n - \theta) \xrightarrow{D} \underline{U} \sim N_g(\underline{0}, \Sigma)$ as $n \rightarrow \infty$,

$\underline{W}_{n,m}^* = \sqrt{m} (T_{n,m}^* - T_n) \xrightarrow{D} \underline{W}_n \sim N_g(\underline{0}, \Sigma_n)$ as $m \rightarrow \infty$

for fixed n , and $\underline{W}_n \xrightarrow{D} \underline{U}$ as $n \rightarrow \infty$.

WRT bootstrap dist, Σ and Σ_n are constant matrices

Hence a) holds.

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The result does not depend on m as

long as $m \rightarrow \infty$. So b) holds. $\square$

(Take $\underline{z}_n \sim N_p(0, \Sigma)$ to mean $\underline{z}_n = \Sigma^{1/2} N_p(0, I_p)$.)

ex) Let $\underline{x}_1, \dots, \underline{x}_n$ be iid ^{random} $p \times 1$ vectors with

$$E(\underline{x}_i) = \underline{\mu} \quad \text{and} \quad \text{cov}(\underline{x}_i) = \Sigma.$$

Then $\sqrt{n}(\bar{\underline{x}}_n - \underline{\mu}) \xrightarrow{D} N_p(0, \Sigma)$ by the MCLT.

a) For the parametric bootstrap

let $\underline{x}_1^*, \dots, \underline{x}_n^* \stackrel{\text{iid}}{\sim} N_p(\bar{\underline{x}}_n, S_n)$ where

^{bootstrap data set}
the sample covariance matrix $S_n \xrightarrow{P} \Sigma$ as $n \rightarrow \infty$.

Then $E(\underline{x}_i^*) = \bar{\underline{x}}_n$ and $\text{cov}(\underline{x}_i^*) = S_n$.

For fixed n , by the MCLT,

$$\sqrt{m}(\bar{\underline{x}}_{nm}^* - \bar{\underline{x}}_n) \xrightarrow{D} N_p(0, S_n) \text{ where}$$

$\bar{\underline{x}}_{nm}^*$ is the sample mean of $\underline{x}_1^*, \dots, \underline{x}_m^*$.

$$\therefore \sqrt{n}(\bar{\underline{x}}_n^* - \bar{\underline{x}}_n) \xrightarrow{D} N_p(0, \Sigma) \text{ by } \boxed{\text{ZZ}}.$$

BPT

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b) For the nonparametric bootstrap

$\underline{x}_1^*, \dots, \underline{x}_n^*$ are iid with $E(\underline{x}_i^*) = \underline{\bar{x}}_n$

and $\text{Cov}(\underline{x}_i^*) = S_m = \frac{n-1}{n} S_n$ by

ex on notes 69.5.

By the MCLT,

$$\sqrt{m} (\bar{X}_{nm}^* - \bar{x}_n) \xrightarrow{D} N_p(0, S_m).$$

$$\therefore \sqrt{n} (\bar{X}_n^* - \bar{x}_n) \xrightarrow{D} N_p(0, \Sigma) \text{ as } n \rightarrow \infty$$

Note! in a) we did not assume that $\underline{x}_1, \dots, \underline{x}_n$ are iid $N_p(\mu, \Sigma)$,

but the parametric bootstrap can be used to get cutoffs since $\underline{\bar{x}}$ has a great CLT. Also if $T = \underline{\bar{x}}$,

$$S_T^* \rightarrow S_n \text{ as } m \rightarrow \infty \text{ for a) and}$$

$$S_T^* \rightarrow S_m \text{ as } m \rightarrow \infty \text{ for b).}$$

ex] $Y_1, \dots, Y_n$ ind $D(\underline{x}_i^T \underline{\beta}, \underline{\tau})$

really $Y_i | \underline{x}_i$

$\uparrow_{p \times 1} \quad \uparrow_{q \times 1} \quad q \geq 0$

and MLE $\sqrt{n}(\hat{\underline{\beta}}_n - \underline{\beta}) \xrightarrow{D} N_p(\underline{0}, \underline{I}_1(\underline{\beta}))$

Parametric bootstrap

$\underline{y}_1^*, \dots, \underline{y}_n^*$ ind $D(\underline{x}_i^T \hat{\underline{\beta}}_n, \hat{\underline{\tau}})$

satisfies above model.

$\therefore \sqrt{m}(\hat{\underline{\beta}}_{m,n}^* - \hat{\underline{\beta}}_n) \xrightarrow{D} N_p(\underline{0}, \underline{I}_1(\hat{\underline{\beta}}_n))$

Since $\underline{I}_1(\hat{\underline{\beta}}_n) \xrightarrow{P} \underline{I}_1(\underline{\beta})$

usual estimator of $\underline{I}_1(\underline{\beta})$

$\sqrt{n}(\hat{\underline{\beta}}_n^* - \hat{\underline{\beta}}_n) \xrightarrow{D} N_p(\underline{0}, \underline{I}_1(\underline{\beta}))$ as $n \rightarrow \infty$.

This bootstrap can be used for GLMs such as Poisson regression

where $Y_i | \underline{x}_i \sim \text{Poisson}[\exp(\underline{x}_i^T \underline{\beta})]$.

See HW 8.

Olive § 5.1 CIs and
confidence regions without the bootstrap
Topic occurs occasionally on M580 goods.

LS 76

23} Suppose $\sqrt{n}(\bar{T}_n - \theta) \xrightarrow{D} N(0, \sigma^2(\theta))$.

Then $\sqrt{n}(\bar{T}_n - \theta) \approx N(0, \sigma^2(\theta))$,

$\therefore \frac{\sqrt{n}(\bar{T}_n - \theta)}{\sigma(\hat{\theta})} \approx N(0, 1)$ where

often $\bar{T}_n = \hat{\theta}$. $\therefore 1 - \delta \approx$

$$P\left(-z_{1-\frac{\delta}{2}} \leq \frac{\sqrt{n}(\bar{T}_n - \theta)}{\sigma(\hat{\theta})} \leq z_{1-\frac{\delta}{2}}\right)$$

$$= P\left(\frac{\sigma(\hat{\theta})}{\sqrt{n}}(-z_{1-\frac{\delta}{2}}) \leq \bar{T}_n - \theta \leq \frac{\sigma(\hat{\theta})}{\sqrt{n}} z_{1-\frac{\delta}{2}}\right)$$

$$= P\left(-\bar{T}_n - \frac{\sigma(\hat{\theta})}{\sqrt{n}} z_{1-\frac{\delta}{2}} \leq -\theta \leq -\bar{T}_n + \frac{\sigma(\hat{\theta})}{\sqrt{n}} z_{1-\frac{\delta}{2}}\right)$$

$$= P\left(\bar{T}_n + z_{1-\frac{\delta}{2}} \frac{\sigma(\hat{\theta})}{\sqrt{n}} \geq \theta \geq \bar{T}_n - z_{1-\frac{\delta}{2}} \frac{\sigma(\hat{\theta})}{\sqrt{n}}\right)$$

$$= P\left(\bar{T}_n - \frac{\sigma(\hat{\theta})}{\sqrt{n}} z_{1-\frac{\delta}{2}} \leq \theta \leq \bar{T}_n + z_{1-\frac{\delta}{2}} \frac{\sigma(\hat{\theta})}{\sqrt{n}}\right)$$

Hence $\left[\bar{T}_n - z_{1-\frac{\alpha}{2}} \frac{\sigma(\hat{\theta})}{\sqrt{n}}, \bar{T}_n + z_{1-\frac{\alpha}{2}} \frac{\sigma(\hat{\theta})}{\sqrt{n}} \right]$

is a large sample $100(1-\alpha)\%$ CI for θ .

24) Often $\sigma^2(\theta) = \frac{1}{I_1(\theta)}$ and $\hat{\theta}$ is the MLE of θ .

25) Make sure any unknown parameters in the "CI" are replaced by

estimators: $\sigma(\theta) \rightarrow \sigma(\hat{\theta})$

$$\sqrt{\frac{1}{I_1(\theta)}} \rightarrow \sqrt{\frac{1}{I_1(\hat{\theta})}}$$

Forgetting the hats is a common error.

see HW 9 #1.

ex) Let $\begin{pmatrix} x_i \\ y_i \end{pmatrix}$ be iid and

let $\hat{\Sigma} = \frac{1}{n} \sum_{i=1}^n \begin{pmatrix} x_i \\ y_i \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix}^T = \widehat{\text{cov}}(\underline{x}, \underline{y})$. Want a

large sample 100(1- δ)% confidence region for Σ .

$$\text{Let } \tilde{\Sigma}_{xy} = \frac{1}{n} \sum_{i=1}^n (\underline{x}_i - \bar{\underline{x}})(\underline{y}_i - \bar{\underline{y}})^T = \tilde{\Sigma},$$

$$\text{and } \hat{\Sigma}_{xy} = \frac{n}{n-1} \tilde{\Sigma}_{xy} = \frac{1}{n-1} \sum_{i=1}^n (\underline{x}_i - \bar{\underline{x}})(\underline{y}_i - \bar{\underline{y}})^T.$$

$$\begin{aligned} \text{Let } \Sigma_{xy} &= \text{cov}(\underline{x}, \underline{y}) = E[(\underline{x} - E(\underline{x}))(\underline{y} - E(\underline{y}))^T] \\ &= E(\underline{x} \underline{y}^T) - E(\underline{x})E(\underline{y})^T = E[(\underline{x} - E(\underline{x}))\underline{y}^T] \\ &= E[\underline{x}(\underline{y} - E(\underline{y}))^T]. \text{ Let } \mu_x = E(\underline{x}) \text{ and } \mu_y = E(\underline{y}). \end{aligned}$$

Let $\underline{w}_i = (\underline{x}_i - \mu_x)(\underline{y}_i - \mu_y)^T$ have sample mean $\bar{\underline{w}}_n$. Let $\Sigma_w = \text{cov}(\underline{w})$. Claim

$$\sqrt{n}(\bar{\underline{w}}_n - \underline{w}) \xrightarrow{D} N_p(0, \Sigma_w)$$

$$\sqrt{n}(\hat{\Sigma}_n - \Sigma) \xrightarrow{D} N_p(0, \Sigma_w), \quad \text{and}$$

$$\sqrt{n} (\hat{\mu}_n - \mu) \xrightarrow{D} N_p(0, \Sigma_w).$$

$$\text{Let } \hat{\Sigma}_w = \hat{\Sigma}_z + O_p(n^{-1/2}) = \frac{1}{n-1} \sum_{i=1}^n (\underline{z}_i - \bar{\underline{z}})(\underline{z}_i - \bar{\underline{z}})^T$$

$$\text{where } \underline{z}_i = \underline{x}_i (y_i - \bar{y}). \text{ Let } \hat{\Sigma}_z = \frac{n-1}{n} \hat{\Sigma}_z.$$

Assume $\text{Cov}(\underline{w}_i)$ exists, hence if

$$\underline{x}_i = (x_{i1}, \dots, x_{ip})^T, \text{ assume } E(x_{ij}^k y_i^m)$$

exist for $j=1, \dots, p$, $k, m=0, 1, 2$.

Proof) Need $\text{Cov}(\underline{w}_i) = E[(\underline{x}_i - \underline{\mu}_x)(\underline{x}_i - \underline{\mu}_x)^T (y_i - \mu_y)^2] = \hat{\Sigma}_{xy} \hat{\Sigma}_{xy}^T$ to exist for $\hat{\Sigma}_z$ to be a

consistent estimator of Σ_w , as will be shown.

$$\text{Now } n \hat{\Sigma}_z = \sum_{i=1}^n (\underline{x}_i - \underline{\mu}_x + \underline{\mu}_x - \bar{\underline{x}})(y_i - \mu_y + \mu_y - \bar{y})$$

$$= \underbrace{\sum_i (\underline{x}_i - \underline{\mu}_x)(y_i - \mu_y)}_{\underline{w}_i} + \sum_i (\underline{x}_i - \underline{\mu}_x)(\mu_y - \bar{y}) + (\underline{\mu}_x - \bar{\underline{x}}) \sum_i (y_i - \mu_y) + n(\underline{\mu}_x - \bar{\underline{x}})(\mu_y - \bar{y})$$

$$= \sum \underline{w}_i + \overbrace{n(\bar{x} - \mu_x)(\bar{y} - \mu_y)}^{-a_n} + \overbrace{n(\mu_x - \bar{x})(\bar{y} - \mu_y)}^{-a_n} \quad L978$$

$$+ \underbrace{n(\mu_x - \bar{x})(\mu_y - \bar{y})}_{\sim n}$$

$$= \sum_i \underline{w}_i - n(\bar{x} - \mu_x)(\bar{y} - \mu_y) = n \tilde{\eta}$$

$$\text{Thus } \sqrt{n} \tilde{\eta} = \sqrt{n} \frac{1}{n} \sum_i \underline{w}_i - \underbrace{\sqrt{n}(\bar{x} - \mu_x)}_{O_p(1)} \underbrace{\sqrt{n}(\bar{y} - \mu_y)}_{O_p(1)}$$

$$\underbrace{\quad}_{\sqrt{n}}$$

$$O_p(1)/\sqrt{n} = O_p(1)$$

$$\therefore \sqrt{n}(\tilde{\eta} - \eta) = \sqrt{n}(\bar{w} - \eta) + O_p(1)$$

where the $\underline{w}_i$ are iid with $E(\underline{w}_i) = \eta = \text{cov}(X, Y)$

and $V = \text{cov}(\underline{w}) = \mathbb{E} \underline{w} = E[(\underline{w} - E(\underline{w}))(\underline{w} - E(\underline{w}))^T]$

$$\therefore \sqrt{n}(\bar{w} - \eta) \xrightarrow{D} N_p(0, \mathbb{E} \underline{w}) \text{ by MCLT}$$

$$\sqrt{n}(\tilde{\eta} - \eta) \xrightarrow{D} N_p(0, \mathbb{E} \underline{w}) \text{ by Slutsky's Th and}$$

$$\sqrt{n}(\hat{\eta} - \eta) \xrightarrow{D} N_p(0, \mathbb{E} \underline{w}).$$

$$\underline{w}_i = (\underline{x}_i - \underline{\bar{x}} + \underline{\bar{x}} - \underline{\mu}_x) (\underline{y}_i - \underline{\bar{y}} + \underline{\bar{y}} - \underline{\mu}_y) =$$

$$\overset{c_1}{\underline{v}_i} + (\underline{x}_i - \underline{\bar{x}}) \overset{c_2}{(\underline{\bar{y}} - \underline{\mu}_y)} + (\underline{\bar{x}} - \underline{\mu}_x) \overset{c_3}{(\underline{y}_i - \underline{\bar{y}})} + (\underline{\bar{x}} - \underline{\mu}_x) \overset{c_4}{(\underline{\bar{y}} - \underline{\mu}_y)}.$$

$$\underline{\bar{w}} = \underline{\bar{v}} + \underbrace{\frac{1}{n} \sum (\underline{x}_i - \underline{\bar{x}}) (\underline{\bar{y}} - \underline{\mu}_y)}_0 + \underbrace{\frac{1}{n} \sum (\underline{\bar{x}} - \underline{\mu}_x) (\underline{y}_i - \underline{\bar{y}})}_0 + (\underline{\bar{x}} - \underline{\mu}_x) (\underline{\bar{y}} - \underline{\mu}_y)$$

$$\underline{\bar{w}} = \underline{\bar{v}} + (\underline{\bar{x}} - \underline{\mu}_x) (\underline{\bar{y}} - \underline{\mu}_y)$$

$$\underline{w}_i - \underline{\bar{w}} = c_1 + c_2 + c_3 + c_4 - \underline{\bar{v}} - c_4$$

$$= \underline{v}_i - \underline{\bar{v}} + c_2 + c_3 = \underline{v}_i - \underline{\bar{v}} + \underline{a}_i$$

$$\text{where } \underline{a}_i = c_2 + c_3 = (\underline{x}_i - \underline{\bar{x}}) (\underline{\bar{y}} - \underline{\mu}_y) + (\underline{\bar{x}} - \underline{\mu}_x) (\underline{y}_i - \underline{\bar{y}})$$

$$= O_p(n^{-1/2}) \text{ since } \sqrt{n} (c_2 + c_3) =$$

$$\underbrace{(\underline{x}_i - \underline{\bar{x}}) \sqrt{n} (\underline{\bar{y}} - \underline{\mu}_y)}_{O_p(1)} + \underbrace{\sqrt{n} (\underline{\bar{x}} - \underline{\mu}_x) (\underline{y}_i - \underline{\bar{y}})}_{O_p(1)} = O_p(1)$$

$$\text{So } \underline{\bar{w}} = \frac{1}{n} \sum_i (\underline{w}_i - \underline{\bar{w}}) (\underline{w}_i - \underline{\bar{w}})^T =$$

$$\frac{1}{n} \sum_i (\underline{v}_i - \underline{\bar{v}} + \underline{a}_i) (\underline{v}_i - \underline{\bar{v}} + \underline{a}_i)^T$$

$$= \frac{1}{n} \sum_i (\underline{v}_i - \underline{\bar{v}}) (\underline{v}_i - \underline{\bar{v}})^T + \underbrace{\frac{1}{n} \sum_i O_p(n^{-1/2})}_{\frac{1}{n} n O_p(n^{-1/2}) = O_p(n^{-1/2})}$$

$$= \underline{\bar{v}} + O_p(n^{-1/2})$$

$$\sqrt{n}(\bar{y} - \mu) \xrightarrow{D} N(0, \sigma^2)$$

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n \tilde{y}_i$$

$$\sqrt{n}(\hat{\mu} - \mu) = \sqrt{n}\left(\frac{1}{n} \sum_{i=1}^n \tilde{y}_i - \mu\right) = \sqrt{n}\left[\frac{1}{n} \sum_{i=1}^n \tilde{y}_i - \frac{1}{n} \sum_{i=1}^n \mu\right]$$

$$= \underbrace{\sqrt{n} \frac{n-1}{n} (\bar{\tilde{y}} - \mu)}_{\rightarrow 0} + \underbrace{\sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n \tilde{y}_i - \mu\right)}_{\rightarrow 0}$$

$$\sqrt{n} \left[\frac{(n-1)/n}{n-1} \right] = \sqrt{n} \left[\frac{1}{n-1} \right] \rightarrow 0$$

Hence if the $\begin{pmatrix} x_i \\ y_i \end{pmatrix}$ are iid

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$$\sqrt{n}(\hat{\underline{m}} - \underline{m}) \sim AN_p(\underline{0}, \hat{\Sigma}_z)$$

$$\text{and } n(\hat{\underline{m}} - \underline{m})^T \hat{\Sigma}_z^{-1} (\hat{\underline{m}} - \underline{m}) \xrightarrow{D} \chi_p^2.$$

$$\sqrt{n} A(\hat{\underline{m}} - \underline{m}) \sim AN_K(\underline{0}, A \hat{\Sigma}_z A^T)$$

$$\text{and } n(A\hat{\underline{m}} - A\underline{m})^T (A \hat{\Sigma}_z A^T)^{-1} (A\hat{\underline{m}} - A\underline{m}) \xrightarrow{D} \chi_K^2$$

where A is full rank K ,
 $K \times p$

To test $H_0: A\underline{m} = \underline{\theta}_0$ use cont. reg.

$$\left\{ \underline{w} : n(A\hat{\underline{m}} - \underline{w})^T (A \hat{\Sigma}_z A^T)^{-1} (A\hat{\underline{m}} - \underline{w}) \leq \chi_{K, \alpha}^2 \right\}$$

or reject H_0 if

$$n(A\hat{\underline{m}} - \underline{\theta}_0)^T (A \hat{\Sigma}_z A^T)^{-1} (A\hat{\underline{m}} - \underline{\theta}_0) \geq \chi_{K, \alpha}^2.$$

Replacing cutoff $\chi_{K, \alpha}^2$ by $K F_{K, n, \alpha}$

where $d_n \rightarrow \infty$ is a good idea.

79.5

could use $d_n = n-1$ or $d_n = n-p$ or $d_n = n-k$.

Back to the bootstrap

could skip
and go to
79.5.2.16 B

26} Geometric Argument Suppose

$$\sqrt{n}(T_n - \theta) \xrightarrow{D} U, \quad \sqrt{n}(T_i^* - T_n) \xrightarrow{D} U$$

where $E(U) = 0$, $\text{Cov}(U) = \Sigma_T > 0$.

Suppose we had an iid sample

$T_1, \dots, T_B$ of the statistic where

$T_i = T_n$. Compute the nonparametric

prediction region R_B on the T_i so

$P(T_i \in R_B) \approx 95\%$. Randomly select

a T_i and center the pred reg at

T_i instead of at $\bar{T}_B$. Then $P(\text{new}$

region contains $\bar{T}_B) \approx 95\%$.