Large Sample
Final review

1) a) CLT (variant: $Y_n = \sum_{i=1}^{n} X_i$, X_i iid)
b) delta method
c) second order delta method
quotient rule, product rule, chain rule

2) Does $X_n \xrightarrow{D} X$?

a) show $F_{X_n}(t) \to F_X(t)$ at continuity points of F_X (if not, X_n does not converge in dist)

b) show $M_{X_n}(t) \to M_X(t)$ $|t| < d$

c) show $C_{X_n}(t) \to C_X(t)$ $\forall t \in \mathbb{R}$

continuity th

d) show $C_{X_n}(t) \to C(t)$
where C is continuous at $t=0$

e) If $C_{X_n}(t) \to C(t)$ and C is not continuous at 0 then X_n does not converge in dist to any RV X.

3) conv in prob, consistent estimators
I) def
II) i) $V_\theta(T_n) \to 0$ and ii) $E_\theta(T_n) \to \tau(\theta)$
III) $MSE_{\tau(\theta)}(T_n) \to 0$ IV) $n^\delta(T_n - \tau(\theta)) \xrightarrow{D} X_\theta$

4) $Y_n \xrightarrow{r} Y$

5) WLLN $\overline{Y}_n \xrightarrow{P} \mu$ $\quad$ Y_i iid, $E(Y_i) = \mu$

SLLN $\overline{Y}_n \xrightarrow{wp1} \mu$.

6) CLT for MED(n) $\quad \sqrt{n}\,(MED(n) - MED(Y)) \xrightarrow{D} N\left(0, \frac{1}{4 f^2(MED(Y))}\right)$

MLE
UMVUE

$$\underbrace{\sqrt{n}\,[T(\hat{\theta}_n) - T(\theta)] \xrightarrow{D} N\left(0, \frac{[T'(\theta)]^2}{I_1(\theta)}\right)}$$

then $T(\hat{\theta})$ is asy efficient

7) $n^{\delta}(T_i - \theta) \xrightarrow{D} N(0, \sigma_i^2(F))$

$$ARE(T_1, T_2) = \frac{\sigma_2^2(F)}{\sigma_1^2(F)}$$

8) Slutsky's th and corollaries

If $Y_n \xrightarrow{A} Y$ and $W_n \xrightarrow{B} c$

then $Y_n + W_n \xrightarrow{D} Y + c$

$Y_n W_n \xrightarrow{D} cY$

$Y_n / W_n \xrightarrow{D} Y/c$ if $c \neq 0$

If $Y = d$, can replace $\xrightarrow{D}$ by $\xrightarrow{P}$.

9) MCLT $\sqrt{n}\,(\underline{\bar{X}}-\underline{\mu}) \xrightarrow{D} N_k(\underline{0}, \Sigma)$

10) mult delta method

If $\sqrt{n}\,(T_n-\underline{\theta}) \xrightarrow{D} N_k(\underline{0}, \Sigma)$

then $\sqrt{n}\,(g(T_n)-g(\theta)) \xrightarrow{D} N_d(\underline{0}, D\Sigma D^T)$

$$D = \begin{bmatrix} \frac{\partial}{\partial\theta_1} g_1(\underline{\theta}) & \cdots & \frac{\partial}{\partial\theta_k} g_1(\underline{\theta}) \\ \vdots & & \\ \frac{\partial}{\partial\theta_1} g_d(\theta) & \cdots & \frac{\partial}{\partial\theta_k} g_d(\underline{\theta}) \end{bmatrix}$$

11) MLE $\sqrt{n}\,(\hat{\underline{\theta}}-\underline{\theta}) \xrightarrow{D} N_k(\underline{0}, I^{-1}(\underline{\theta}))$

12) $\underline{x_n} \xrightarrow{D} N_k(\underline{\mu}, \Sigma) \Rightarrow$

$\underline{a} + b\,\underline{x_n} \xrightarrow{D} N_k(\underline{a} + b\underline{\mu}, b^2\Sigma)$

$\underset{\substack{\uparrow \\ q\times k}}{A}\,\underline{x_n} + \underline{d} \xrightarrow{D} N_q(A\underline{\mu}+\underline{d}, A\Sigma A^T)$

13) continuity th $\underline{x_n} \xrightarrow{D} \underset{k\times 1}{\underline{x}}$ iff

$c_{\underline{x_n}}(\underline{t}) \to c_{\underline{x}}(\underline{t}) \quad \forall\ \underline{t} \in \mathbb{R}^k$

14) Cramér Wold device

$\underset{k\times 1}{\underline{X}_n} \xrightarrow{D} \underline{X}$ iff $\underline{t}^T \underline{X}_n \xrightarrow{D} \underline{t}^T \underline{X}$ $\forall \underline{t} \in \mathbb{R}^k$

15) continuous mapping th. If $\underset{k\times 1}{\underline{X}_n \xrightarrow{D} \underline{X}}$ and $\underline{g} : \mathbb{R}^k \to \mathbb{R}^j$ is continuous, then

$\underline{g}(\underline{X}_n) \xrightarrow{D} \underline{g}(\underline{X})$.

16) $\underset{k\times 1}{\underline{X}_n \xrightarrow{D} \underline{X}} \Rightarrow X_{in} \xrightarrow{D} X_i \qquad i=1,\dots,k$

17) If $\underline{X}_{1n}, \underline{X}_{2n}, \dots, \underline{X}_{kn}$ are ind and $\underline{X}_{in} \xrightarrow{D} \underline{X}_i$, then

$$\begin{bmatrix} \underline{X}_{1n} \\ \vdots \\ \underline{X}_{kn} \end{bmatrix} \xrightarrow{D} \begin{bmatrix} \underline{X}_1 \\ \vdots \\ \underline{X}_k \end{bmatrix}$$ where $\underline{X}_1, \dots, \underline{X}_k$ are ind.

18) Quantile th $\sqrt{n}\left((\hat{\xi}_{\alpha_1}, \dots, \hat{\xi}_{\alpha_k})^T - (\xi_{\alpha_1}, \dots, \xi_{\alpha_k})^T\right)$

$\xrightarrow{D} N_k(\underline{0}, \Sigma)$, $\Sigma = (\sigma_{ij})$, $\sigma_{ij} = \dfrac{\alpha_i(1-\alpha_j)}{f(\xi_{\alpha_i})\, f(\xi_{\alpha_j})}$.

19) $\underline{U}_n \xrightarrow{D} \underline{U}$, $C_n^{-1} \xrightarrow{P} C^{-1}$ $\Rightarrow$

$$D_n^2 = \underline{U}_n^T C_n^{-1} \underline{U}_n \xrightarrow{D} D^2 = \underline{U}^T C^{-1} \underline{U}$$

20) CI technique; $\sqrt{n}(T_n - \theta) \xrightarrow{D} N(0, \sigma^2(\theta))$

$$\Rightarrow \left[T_n - z_{1-\frac{\delta}{2}} \frac{\sigma(\hat{\theta})}{\sqrt{n}}, T_n + z_{1-\frac{\delta}{2}} \frac{\sigma(\hat{\theta})}{\sqrt{n}}\right]$$

is a large sample $100(1-\delta)\%$ CI for θ where $\sigma(\hat{\theta})$ is a consistent est of $\sqrt{\sigma^2(\theta)}$.

21) OLS CLT a) $\frac{X^T X}{n} \rightarrow W^{-1}$, $\underline{Y} = X\underline{\beta} + \underline{e}$

$$\sqrt{n}(\hat{\underline{\beta}}_{OLS} - \underline{\beta}) \xrightarrow{D} N_p(\underline{0}, \sigma^2 W)$$

b) $\underline{Z} = W \underline{\eta} + \underline{e}$, $\frac{W^T W}{n} \xrightarrow{P} V^{-1}$

$$\sqrt{n}(\hat{\underline{\eta}}_{OLS} - \underline{\eta}) \rightarrow N_{p-1}(\underline{0}, \sigma^2 V).$$