Math 583   HW 3 Fall 2020   Due Friday, Sept. 11.
BRING 3 SHEETS OF NOTES AND CALCULATOR TO EVERY QUIZ.
Quiz 3 on Wednesday will have problems like HW2 2.14b and HW3 10.1.

Note that Exam 1 is Friday Sept. 18 and Exam 2 is now Friday Oct. 23.

Problem numbers are currently from Olive (2008). These are 3.1, 3.2, 3.6, and 3.9 from Olive (2020).

A 10.1. Suppose that

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \sim N_4 \left(\begin{pmatrix} 49 \\ 100 \\ 17 \\ 7 \end{pmatrix}, \begin{pmatrix} 3 & 1 & -1 & 0 \\ 1 & 6 & 1 & -1 \\ -1 & 1 & 4 & 0 \\ 0 & -1 & 0 & 2 \end{pmatrix} \right).$$

a) Find the distribution of X_2.
b) Find the distribution of $(X_1, X_3)^T$.
c) Which pairs of random variables X_i and X_j are independent?
d) Find the correlation $\rho(X_1, X_3)$.

B) 10.2 Recall that if $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then the conditional distribution of $\boldsymbol{X}_1$ given that $\boldsymbol{X}_2 = \boldsymbol{x}_2$ is multivariate normal with mean $\boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\boldsymbol{x}_2 - \boldsymbol{\mu}_2)$ and covariance matrix $\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$.

Let $\sigma_{12} = \text{Cov}(Y, X)$ and suppose Y and X follow a bivariate normal distribution

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N_2 \left(\begin{pmatrix} 49 \\ 100 \end{pmatrix}, \begin{pmatrix} 16 & \sigma_{12} \\ \sigma_{12} & 25 \end{pmatrix} \right).$$

a) If $\sigma_{12} = 0$, find $Y|X$. Explain your reasoning.

b) If $\sigma_{12} = 10$ find $E(Y|X)$.

c) If $\sigma_{12} = 10$, find $\text{Var}(Y|X)$.

crancap	hdlen	hdht	Data for 10.6
1485	175	132	
1450	191	117	
1460	186	122	
1425	191	125	
1430	178	120	
1290	180	117	
90	75	51	

C) 10.6. The table ($\boldsymbol{W}$) above represents 3 head measurements on 6 people and one ape. Let X_1 = *cranial capacity*, X_2 = *head length* and X_3 = *head height*. Let $\boldsymbol{x} = (X_1, X_2, X_3)^T$. Several multivariate location estimators, including the coordinatewise median and sample mean, are found by applying a univariate location estimator to each random variable and then collecting the results into a vector. a) Find the coordinatewise median MED($\boldsymbol{W}$). (Find the sample mean of each column.)

b) Find the sample mean $\overline{\boldsymbol{x}}$. (Finde the sampe median of each column.)

D) 10.9. Suppose $\boldsymbol{Y} \sim N_n(\boldsymbol{X\beta}, \sigma^2\boldsymbol{I})$. Find the distribution of $(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y}$ if $\boldsymbol{X}$ is an $n \times p$ full rank constant matrix.