

583

1) Consider the Cushny and Peebles data set listed below.

0.0 0.8 1.0 1.2 1.3 1.3 1.4 1.8 2.4 4.6

5 a) Find the sample median MED(n).

1.3

5 b) Find the sample median absolute deviation MAD(n).

-1.3 -1.5 -3 -1 0 0 .1 .5 1.1 3.3
0 0 .1 .1 .3 .5 .5 1.1 1.3 3.3

$\frac{.3 + .5}{2} = 0.4$

Parts c)-f) refer to the CI based on MED(n).

c) Find L_n .

$\lfloor \frac{n}{2} \rfloor - \lceil \sqrt{\frac{n}{4}} \rceil = 5 - \lceil 1.58 \rceil = 5 - 2 = 3$

5 d) Find U_n . $= n - L_n = 7$

e) Find the degrees of freedom p . $U_n - L_n - 1 = 7 - 3 - 1 = 3$

5 f) Find SE(MED(n)).

$\frac{Y_{(n)} - Y_{(L_n+1)}}{2} = \frac{Y_{(7)} - Y_{(4)}}{2} = \frac{1.4 - 1.2}{2} = 0.1$

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2.13

2) Consider the following data set on Math 580 homework scores.

66.7 76.0 | 89.7 90.0 94.0 94.0 95.0 95.3 | 97.0 97.7

Consider the CI based on the 25% trimmed mean.

10 a) Find the 25% trimmed mean T_n . $= \frac{89.7 + \dots + 95.3}{6} = \frac{558}{6} = \boxed{93.0}$

5 a) b) Find L_n . $= \left\lfloor \frac{n}{4} \right\rfloor = \left\lfloor \frac{10}{4} \right\rfloor = \left\lfloor 2.5 \right\rfloor = \boxed{2}$

5 c) Find U_n . $= n - L_n = 10 - 2 = \boxed{8}$

5 d) Find the degrees of freedom p $U_n - L_n - 1 = 8 - 2 - 1 = \boxed{5}$

5 e) Find d_1, \dots, d_{10} .

89.7, 89.7, 89.7, 90, 94, 94, 95, 95.3, 95.3, 95.3

5 f) Find \bar{d} . $= \frac{\sum d_i}{n} = \frac{928}{10} = \boxed{92.8}$

5 g) Find $S^2(d_1, \dots, d_{10})$. $= \frac{\sum d_i^2 - n(\bar{d})^2}{n-1} = \frac{86181.54 - 10(92.8)^2}{9} = \frac{63.14}{9} = \boxed{7.01556}$

10 e) Find $SE(T_n)$.

$$V_{9w} = \frac{S^2(d_1, \dots, d_{10})}{\left[\frac{U_n - L_n}{n} \right]^2} = \frac{7.01556}{\left(\frac{8-2}{10} \right)^2} = 19.4877$$

50 $SE(T_n) = \sqrt{\frac{V_{9w}}{n}} = \sqrt{\frac{19.4877}{10}} = \boxed{1.3960}$

3) Suppose that Y is a random variable that is symmetric about 0 with cdf

$$F(y) = \frac{y + \theta}{2\theta}$$

for $-\theta \leq y \leq \theta$ where the parameter $\theta > 0$.

a) What is the population median $MED(Y)$? Explain briefly.

0 by symmetry or since $F(0) = \frac{1}{2}$

b) Find U such that $F(U) = 0.75$.

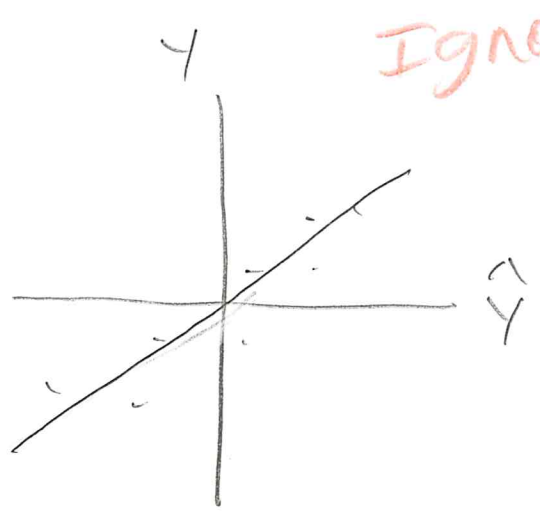
$$\frac{3}{4} = \frac{U + \theta}{2\theta} \text{ or } \frac{3}{2}\theta = U + \theta \text{ or } U = \frac{\theta}{2}$$

c) Find the population median absolute deviation $MAD(Y)$.

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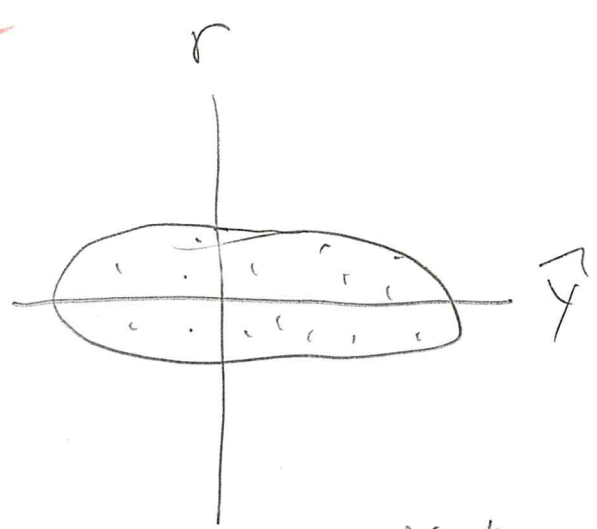
$$U - M = \frac{\theta}{2} - 0 = \frac{\theta}{2}$$

4) Make a rough sketch (drawing) of the two plots that should be made with any multiple linear regression analysis. Place the name of each plot below each sketch.



response plot

IGNORE

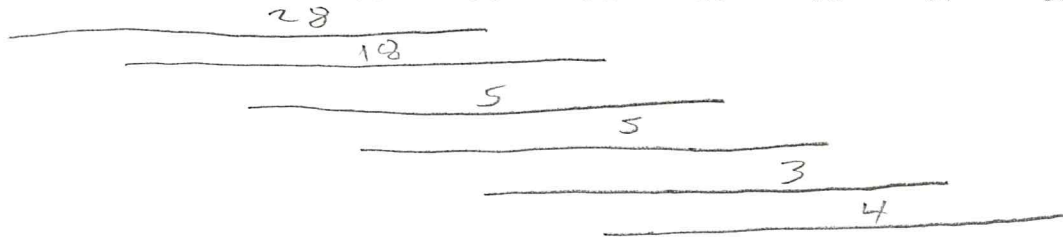


residual plot

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2) Find shorth(5) for the following data set. Show work.

66 76 90 90 94 94 95 95 97 98



$$\text{Shorth}(5) = [94, 97]$$

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e 1) Suppose that

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \sim N_4 \left(\begin{pmatrix} 49 \\ 25 \\ 9 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 & -1 & 3 & 0 \\ -1 & 5 & -3 & 0 \\ 3 & -3 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \right).$$

AI Q5

b a) Find the distribution of X_2 .

$$\sim N(25, 5)$$

b b) Find the distribution of $(X_1, X_3)^T$.

$$\sim N_2 \left[\begin{pmatrix} 49 \\ 9 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix} \right]$$

b c) Which pairs of random variables X_i and X_j are independent?

$$X_1 \perp X_4, \quad X_2 \perp X_4, \quad X_3 \perp X_4$$

b d) Find the correlation $\rho(X_1, X_3)$.

$$\frac{\text{COV}(X_1, X_3)}{\sqrt{\text{Var}(X_1) \text{Var}(X_3)}} = \frac{3}{\sqrt{2} \sqrt{5}} = \frac{3}{\sqrt{10}} = .949$$