

Math 583	old	Quiz 11	Name_____
Coefficient Estimates Full Model Poisson Regression			
Label	Estimate	Std. Error	Est/SE p-value
Constant	-0.406023	0.877382	-0.463 0.6435
bombload	0.165425	0.0675296	2.450 0.0143
exper	-0.0135223	0.00827920	-1.633 0.1024
type	0.568773	0.504297	1.128 0.2594

Coefficient Estimates Reduced Model Poisson Regression			
Label	Estimate	Std. Error	Est/SE p-value
Constant	-1.70097	0.506773	-3.356 0.0008
bombload	0.231122	0.0467598	4.943 0.0000

1) The response  $Y$  is the *number* of locations where aircraft was damaged. The output is from a Poisson regression. The variable *exper* = total months of aircrew experience while *type* of aircraft was coded as 0 or 1. Using the reduced model, find  $\hat{\mu}(x)$  if  $x = \text{bombload} = 8$ .

$$ESP = \beta^T x = \hat{\beta}_1 + \hat{\beta}_2 x_2 = -1.70097 + 0.231122(8) = 0.1480$$

$$\hat{\mu} = e^{ESP} = e^{0.148} = \boxed{1.1595}$$

Logistic Regression Output

Response = y, Terms = (size xray)	Estimate	Std. Error	Est/SE	p-value
Constant	-2.04463	0.609684	-3.354	0.0008
size	1.58827	0.699810	2.270	0.0232
xray	2.11944	0.746659	2.839	0.0045

Number of cases: 53, Degrees of freedom: 50  
Deviance: 53.353

2) In a prostate cancer study, let the response variable  $y = \text{nodal involvement} = 0$  for if absent and 1 if present. Let  $x_1 = \text{size} = 0$  for small, 1 for large. Let  $x_2 = \text{xray} = 0$  for negative, 1 for positive. Logistic regression is used. Predict  $\hat{p}(x)$  if  $\text{size} = x_2 = 1$  and  $\text{xray result} = x_3 = 1$ .

$$ESP = \beta^T x = \hat{\beta}_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_3 = -2.04463 + 1.58827(1) + 2.11944(1) = 1.6631$$

$$\exp(ESP) = \exp(1.6631) = \underline{5.2755} = e^{1.6631}$$

$$\hat{p} = \frac{\exp(ESP)}{1 + \exp(ESP)} = \frac{5.2755}{1 + 5.2755} = \boxed{0.8407}$$

3) The GAM analog to the binary logistic regression model is  $Y_1, \dots, Y_n$  are independent with

$$Y|AP \sim \text{binomial}(1, \rho(AP)) \text{ where } P(\text{success}|AP) = \rho(AP) = \frac{\exp(AP)}{1 + \exp(AP)}.$$

Then

$$\hat{\rho}(\mathbf{x}) = \hat{\rho}(AP) = \rho(EAP) = \frac{\exp(EAP)}{1 + \exp(EAP)}.$$

If  $\mathbf{x}$  is such that  $EAP = 2$ , find  $\hat{\rho}(\mathbf{x})$ .

$$\hat{\rho} = \frac{e^2}{1+e^2} = \frac{7.3891}{8.3891} = 0.8808$$

4) The GAM analog to the Poisson regression model is  $Y_1, \dots, Y_n$  are independent with

$$Y|AP \sim \text{Poisson}(\exp(AP)).$$

Then  $\hat{Y} = E(Y|\mathbf{x}) = E(Y|AP) = \mu(\mathbf{x}) = \exp(AP)$  and

$$\hat{\mu}(\mathbf{x}) = \hat{\mu}(AP) = \exp(EAP).$$

If  $\mathbf{x}$  is such that  $EAP = 3$ , find  $\hat{\mu}(\mathbf{x})$ .

$$\hat{\mu} = e^3 = 20.0855$$