

01004

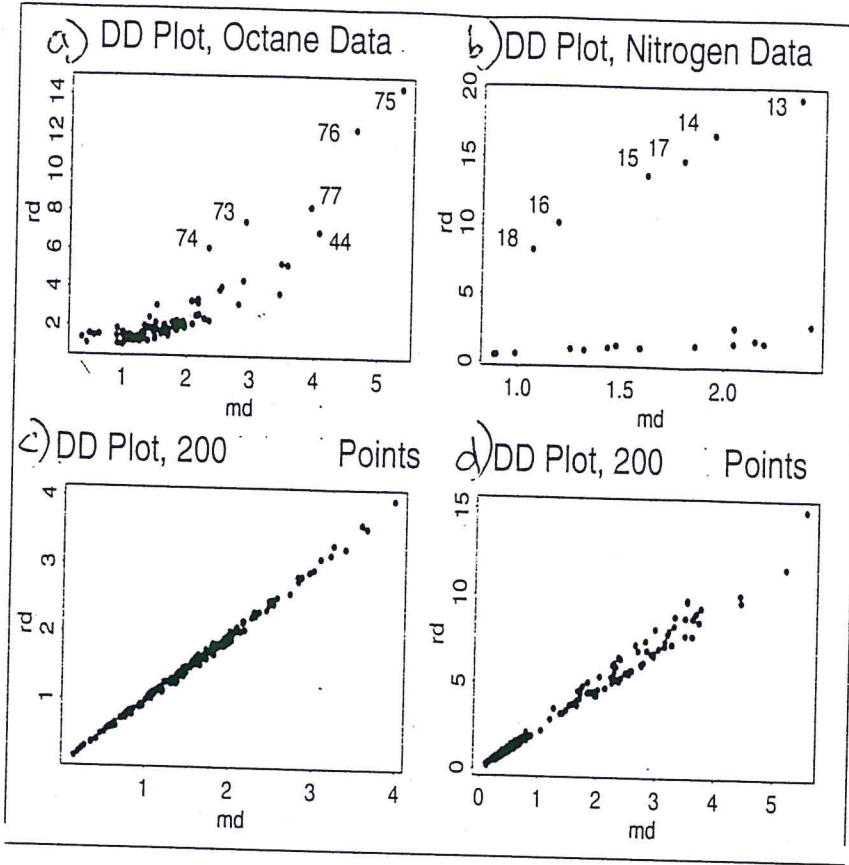


Figure 15.1: 4 DD Plots

1) Shown above are 4 DD plots. Classify the data distribution as multivariate normal, elliptically contoured but not multivariate normal, or not elliptically contoured. Explain your choices briefly.

for each

- a) not EC } do not follow a line with high corr
- b) not EC } (or because of outliers)
- c) MVN follows identity line
- d) EC not MVN: follows a line with slope > 1

490  
07  
F11

5/18 got it completely correct

60

2) Recall that if  $\mathbf{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ , then the conditional distribution of  $\mathbf{X}_1$  given that  $\mathbf{X}_2 = \mathbf{x}_2$  is multivariate normal with mean  $\boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\mathbf{x}_2 - \boldsymbol{\mu}_2)$  and covariance  $\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$ .

Let  $\sigma_{12} = \text{Cov}(Y, X)$  and suppose  $Y$  and  $X$  follow a bivariate normal distribution

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N_2 \left( \begin{pmatrix} 15 \\ 20 \end{pmatrix}, \begin{pmatrix} 64 & \sigma_{12} \\ \sigma_{12} & 81 \end{pmatrix} \right).$$

a) If  $\sigma_{12} = 10$  find  $E(Y|X)$ .

$$\begin{aligned} 15 + 10 \frac{1}{81} (X - 20) &= 12.531 + \frac{10}{81} X \\ &= \underline{12.531 + 0.123 X} \end{aligned}$$

15 ←  
0.123 ←

b) If  $\sigma_{12} = 10$ , find  $\text{Var}(Y|X)$ .

$$64 - 10 \frac{1}{81} 10 = 64 - \frac{100}{81} = \boxed{62.765}$$

c) If  $\sigma_{12} = 10$ , find  $\rho(Y, X)$ , the correlation between  $Y$  and  $X$ .

$$\frac{\sigma_{12}}{\sigma_1 \sigma_2} = \frac{10}{\sqrt{64} \sqrt{81}} = \frac{10}{72} = \boxed{0.1389}$$

see  
F11  
400 final

79