Math 584 HW 1 Spring 2021, due Thursday, January 28.

1) Show that $\boldsymbol{I} - \boldsymbol{H} = \boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$ is idempotent, that is, show that $(\boldsymbol{I} - \boldsymbol{H})(\boldsymbol{I} - \boldsymbol{H}) = (\boldsymbol{I} - \boldsymbol{H})^2 = \boldsymbol{I} - \boldsymbol{H}$.

2) Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be matrices with the same number of rows. If $\boldsymbol{C}$ is another matrix such that $\boldsymbol{A} = \boldsymbol{B}\boldsymbol{C}$, is it true that $rank(\boldsymbol{A}) = rank(\boldsymbol{B})$? Prove or give a counterexample.

3) If $\boldsymbol{A}$ and $\boldsymbol{A}\boldsymbol{B}$ are full rank matrices, is $\boldsymbol{B}$ a full rank matrix? Prove or give a counterexample.

4) Let $\boldsymbol{B} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ -1 & 0 & -2 \\ 1 & 2 & 0 \end{bmatrix}$ and $\boldsymbol{B}^- = \frac{1}{66} \begin{bmatrix} 6 & -2 & -6 & 10 \\ 0 & -11 & 0 & 22 \\ 12 & 7 & -12 & -2 \end{bmatrix}$.

a) Show $(\boldsymbol{B}\boldsymbol{B}^-)' = \boldsymbol{B}\boldsymbol{B}^-$.

b) Show $(\boldsymbol{B}^-\boldsymbol{B})' = \boldsymbol{B}^-\boldsymbol{B}$.

c) Show $\boldsymbol{B}\boldsymbol{B}^-\boldsymbol{B} = \boldsymbol{B}$.

d) Show $\boldsymbol{B}^-\boldsymbol{B}\boldsymbol{B}^- = \boldsymbol{B}^-$.

5) Let $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

a) Explain why $\mathcal{C}(\boldsymbol{A}) = \mathcal{C}(\boldsymbol{B})$.

b) Find $rank(\boldsymbol{A})$.

c) Find a basis for $\mathcal{C}(\boldsymbol{B})$. (Recall that a basis for a space is a linearly independent spanning set for the space.)

d) Find $[\mathcal{C}(\boldsymbol{B})]^\perp$ = nullspace of $\boldsymbol{B}'$.