Math 584 HW 10 Spring 2021, due Thursday, April 8.

1) (Seber and Lee 8a 2 on p. 196)
Show $SSTO = SSE + SSTR$, i.e., show

$$\sum_i \sum_j (Y_{ij} - \overline{Y}_{00})^2 = \sum_i \sum_j (Y_{ij} - \overline{Y}_{i0})^2 + \sum_i \sum_j (\overline{Y}_{i0} - \overline{Y}_{00})^2.$$

2) (Seber and Lee 8a 4 on p. 196)
Let $n = J_1 + \cdots J_I$ with $J_i = n_i$ for $i = 1, ..., I$. a) Prove

$$\sum_i \sum_j (\overline{Y}_{i0} - \overline{Y}_{00})^2 = \sum_i \frac{Y_{i0}^2}{J_i} - \frac{Y_{00}^2}{n}.$$

b) Prove

$$\sum_i \sum_j (Y_{ij} - \overline{Y}_{i0})^2 = \sum_i \sum_j Y_{ij}^2 - \sum_i \frac{Y_{i0}^2}{J_i}.$$

3) (Seber and Lee 11b 5 on p. 352)
Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be orthogonal matrices of the same dimension. Hence $\boldsymbol{A}^{-1} = \boldsymbol{A}^T$ and $\boldsymbol{B}^{-1} = \boldsymbol{B}^T$. Show that $\boldsymbol{C} = \boldsymbol{AB}$ is an orthogonal matrix, i.e. show $(\boldsymbol{AB})^T = (\boldsymbol{AB})^{-1}$.

4) (3.10)
Note that $C(\boldsymbol{X}'\boldsymbol{X}) = C(\boldsymbol{X}')$ since $C(\boldsymbol{X}'\boldsymbol{X}) \subseteq C(\boldsymbol{X}')$ and $rank(\boldsymbol{X}'\boldsymbol{X}) = rank(\boldsymbol{X}')$.

Use this result to explain why there is always a solution $\hat{\boldsymbol{\beta}}$ to the normal equations:

$$\boldsymbol{X}'\boldsymbol{X}\hat{\boldsymbol{\beta}} = \boldsymbol{X}'\boldsymbol{Y}.$$