Math 584 HW 5 Spring 2021, due Thursday, March 4.

Chapters 1-9 of my online notes (http://parker.ad.siu.edu/Olive/regbk.htm) are what I use for Math 484 (Regression and Design). Chapter 2 covers multiple linear regression, the Partial F test, the Anova F test and the Wald t test. Ch. 4 covers WLS and GLS.

The URL (http://parker.ad.siu.edu/Olive/sref.pdf) has some references for Math and Stat texts for classes such as Math 584 Linear Models (this class), Math 221 Linear Algebra, Math 484, and Design of Experiments.

1) Direct maximization of the likelihood equation is easier. From p. 49,

$$L(\boldsymbol{\beta}, \sigma^2) = (2\pi\sigma^2)^{-n/2} \exp\left(\frac{-1}{2\sigma^2}\|\boldsymbol{y} - \boldsymbol{X\beta}\|^2\right).$$

a) Since the least squares estimator $\hat{\boldsymbol{\beta}}$ minimizes $\|\boldsymbol{y} - \boldsymbol{X\beta}\|^2$, show that $\hat{\boldsymbol{\beta}}$ is the MLE of $\boldsymbol{\beta}$.

b) Then find the MLE $\hat{\sigma}^2$ of σ^2 by maximizing

$$L(\sigma^2) \equiv L(\hat{\boldsymbol{\beta}}, \sigma^2) = (2\pi\sigma^2)^{-n/2} \exp\left(\frac{-1}{2\sigma^2}\|\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}\|^2\right).$$

Hint: use the log likelihood and show that the second derivative of the log likelihood evaluated at $\hat{\sigma}^2$ is negative. Also this problem was done in class as 24) for Section 3.5.

2) (Seber 3b 3 on p. 44) Let $Y_1, ..., Y_n$ be a random sample from $N(\theta, \sigma^2)$. Find the linear unbiased estimator of θ with minimum variance.

3) (Seber3c 1a on p. 47.) Suppose $\boldsymbol{Y} \sim N_n(\boldsymbol{X\beta}, \sigma^2\boldsymbol{I}_n)$, where $\boldsymbol{X}$ is $n \times p$ of full rank p. Find $Var(S^2) = Var(MSE)$.

Let $\boldsymbol{R} = \boldsymbol{I} - \boldsymbol{P}$. Use equation (3.14): $Var(\boldsymbol{Y}'\boldsymbol{RY}) = \sigma^4\gamma_2\boldsymbol{r}'\boldsymbol{r} + 2\sigma^4(n-p)$ and the fact that for normal data, $\gamma_2 = 0$.