

1) (Seber 3h 1 on p. 64.) Suppose that  $\mathbf{X}$  does not have full rank, and let  $\hat{\beta}_i$  ( $i = 1, 2$ ) be any two solutions of the normal equations. Show directly that  $\|\mathbf{Y} - \mathbf{X}\hat{\beta}_1\|^2 = \|\mathbf{Y} - \mathbf{X}\hat{\beta}_2\|^2$ .

2) (Seber 3i 1 on p. 65.) Prove that  $\mathbf{a}'E(\hat{\beta})$  is an estimable function of  $\beta$ .  
Hint: 47)-49) on Exam 2 review may be useful.

3) (Seber 3i 2 on p. 65.) If  $\mathbf{a}'_1\beta, \mathbf{a}'_2\beta, \dots, \mathbf{a}'_k\beta$  are estimable, prove than any linear combination of these is also estimable.

4) (Seber 3i 6 on p. 65.) Assume  $\mathbf{X}$  is  $n \times p$ . Prove that all linear functions  $\mathbf{a}'\beta$  are estimable if and only if the columns of  $\mathbf{X}$  are linearly independent.

5) (Seber 4c 3b on p. 113.) Given the full rank model, suppose we wish to test  $H : \beta_j = 0, j \neq 0$ . Let  $R_H^2$  be the coefficient of determination for the model with  $\beta_j = 0$ . Then the  $F$ -statistic for testing  $H$  is

$$F = \frac{R^2 - R_H^2}{1 - R^2} \frac{n - p}{1}.$$

Using  $F \geq 0$ , show that  $R^2$  can never increase when a  $\beta$  coefficient is set equal to zero.  
Hint: Show  $R_H^2 \leq R^2$ .

6) "2.40" Generalized and weighted least squares are each equivalent to a least squares regression without intercept. Let  $\mathbf{V} = \text{diag}(1, 1/2, 1/3, \dots, 1/9) = \text{diag}(W_i)$  where  $n = 9$  and the weights  $w_i = i$  for  $i = 1, \dots, 9$ . Let  $\mathbf{x}' = (1, x_1, x_2, x_3)$ . Then the weighted least squares with weight vector  $\mathbf{w}' = (1, 2, \dots, 9)$  should be equivalent to the OLS regression of  $\sqrt{w_i}y_i = z_i$  on  $\mathbf{u}$  where  $\mathbf{u} = \sqrt{w_i}\mathbf{x} = (\sqrt{w_i}, \sqrt{w_i}x_1, \sqrt{w_i}x_2, \sqrt{w_i}x_3)'$ . There is no intercept because the vector of ones has been replaced by a vector of the  $\sqrt{w_i}$ 's. Type the following commands in *R* and include the output from both *lsfit* commands. The coefficients from both *lsfit* commands should be the same. Print out one page of output, perhaps by copying and pasting the output into *Word* or *Notepad*. You can also email me the one page of output.

The commands can also be copied and pasted into *R* from the file of *R* homework commands for this class at (<http://parker.ad.siu.edu/Olive/linmodrhw.txt>).

```
e <- rnorm(9)
x <- matrix(rnorm(27),nrow=9,ncol=3)
sqrtv <- sqrt(diag(1/1:9))
y <- 4 + x%%c(1,2,3) + sqrtv%%e
wtt <- 1:9
lsfit(x,y,wtt)$coef
kinv <- sqrt(diag(1:9))
z <- kinv%%y
B <- 1 + 0*1:9
B <- cbind(B,x)
```

```
B <- kinv%*%B
lsfit(B,z,int=F)$coef
```

The computer lab is in Neckers 258

computer lab login: If necessary, hit a key such as Ctrl, enter your AD\siu8... (dawg tag) and your password. Left click the lower left icon to see programs. If you are using a computer in the lab for the first time in several months, it takes about 5 minutes to set up your applications, and this will occur for each computer you use, so you may want to use the same computer when you can.

*R* is on computers 11–25. You could Google “David Olive”, get to my personal website, click on “Spring 2021 Math 584-Linear Models webpage” to get the webpage for this course. The link on the bottom takes you to the online book and to the links for *R* homework. My office is right by the computer lab if you need help.

*R* is free software, and the following URL, chapter 11 of the course notes, has more information about *R*. (<http://parker.ad.siu.edu/Olive/linmodch11.pdf>)

*R* is the free software available from the **CRAN** website (<https://cran.r-project.org/>).

Many of the homework problems use *R* functions contained in the book’s website (<http://parker.ad.siu.edu/Olive/linmodbk.htm>) under the file name *linmodpack.txt*. The following two *R* commands can be copied and pasted into *R* from near the top of the file (<http://parker.ad.siu.edu/Olive/linmodrhw.txt>).

**Downloading the book’s *R* functions** *linmodpack.txt* and data files *linmoddata.txt* into *R*: the commands

```
source("http://parker.ad.siu.edu/Olive/linmodpack.txt")
source("http://parker.ad.siu.edu/Olive/linmoddata.txt")
```

can be used to download the *R* functions and data sets into *R*. Type *ls()*. Nearly 10 *R* functions from *linmodpack.txt* should appear. In *R*, enter the command *q()*. A window asking “*Save workspace image?*” will appear. Click on *No* to remove the functions from the computer (clicking on *Yes* saves the functions in *R*, but the functions and data are easily obtained with the source commands).

Becker, R.A., Chambers, J.M., and Wilks, A.R. (1988), *The New S Language A Programming Environment for Data Analysis and Graphics*, Wadsworth and Brooks/Cole, Pacific Grove, CA.

Crawley, M.J. (2005), *Statistics an Introduction Using R*, John Wiley and Sons, Hoboken, NJ.

Crawley, M.J. (2007), *The R Book*, John Wiley and Sons, Hoboken, NJ.