Math 584 HW 9 Spring 2021, due Thursday, April 1. 5) and 6) are good for the quiz 8. Final: Tuesday, May 4, 2:45-4:45.

1) (Seber 9b 2 on p. 232:) When $Cov(\boldsymbol{\epsilon}) = \sigma^2\boldsymbol{V}$, the appropriate estimate of $\boldsymbol{\beta}$ is the generalized least squares estimator $\boldsymbol{\beta}_* = (\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{Y}$. If $C(\boldsymbol{V}^{-1}\boldsymbol{X}) = C(\boldsymbol{X})$, show that $\boldsymbol{\beta}_*$ and $\hat{\boldsymbol{\beta}}$ are identical. Hint: Let $\boldsymbol{Y} = \boldsymbol{Y}_1 + \boldsymbol{Y}_2$ where $\boldsymbol{Y}_1 \in C(\boldsymbol{X})$ and $\boldsymbol{Y}_2 \perp C(\boldsymbol{X})$. So $\boldsymbol{Y}_1 = \boldsymbol{X}\boldsymbol{a}$ for some vector $\boldsymbol{a}$. Show $\boldsymbol{\beta}_* = \hat{\boldsymbol{\beta}} = \boldsymbol{a}$.

2) (Seber 9d 5 on p. 262 with better notation:) a) Prove

$$\frac{\partial}{\partial x_i}\sum_{j=1}^{n}(x_j - \overline{x})(Y_j - \overline{Y}) = Y_i - \overline{Y}.$$

b) Prove $\frac{\partial}{\partial x_i}\sum_{j=1}^{n}(x_j - \overline{x})^2 = 2(x_i - \overline{x})$.

3) (Seber 12a 1 on p. 399:) Let $\boldsymbol{\mu} = E(\boldsymbol{Y})$. Show that the square of the total bias $\|\boldsymbol{\mu} - E[\boldsymbol{X}\hat{\boldsymbol{\beta}}]\|^2 = \boldsymbol{\mu}'(\boldsymbol{I}_n - \boldsymbol{P})\boldsymbol{\mu}$.

4) This problem proves that $\boldsymbol{X}\boldsymbol{\beta} \in \mathcal{C}(\boldsymbol{X})$. Suppose that $\boldsymbol{X} = [\boldsymbol{x}_1, ..., \boldsymbol{x}_p]$ is an $n \times p$ matrix and that $\boldsymbol{\beta} = (\beta_1, ..., \beta_p)'$ is a $p \times 1$ vector. Show that $\boldsymbol{X}\boldsymbol{\beta} = \sum_{i=1}^{p}\beta_i\boldsymbol{x}_i$.

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Output for problem 5 a).
Current terms: (finger to ground nasal height sternal height)
                          df   RSS          |   k    C_I
Delete: nasal height      73   35567.2      |   3    1.617
Delete: finger to ground  73   36878.8      |   3    4.258
Delete: sternal height    73   186259.      |   3  305.047
```

5) a) From the output from backward elimination given above, what terms should be used in the MLR model to predict Y? (You can tell that the nontrivial variables are finger to ground, nasal height and sternal height from the "delete lines." DON'T FORGET THE CONSTANT!) b) What terms (variables) should be used in the model selected from the forward selection output below?

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output for problem 5b)
Base terms: (log[NONW] EDUC log[SO] PREC log[NOX])
               df   RSS          |   k    C_I
Add: JANT      53    58871.      |   7    7.353
Add: log[HC]   53   69233.3      |   7   16.744
Add: HOUS      53   70774.1      |   7   18.141
Add: POPN      53   71424.7      |   7   18.730
Add: POOR      53   72049.4      |   7   19.296
Add: OVR65     53   72337.1      |   7   19.557
Add: JULT      53   72348.6      |   7   19.568
Add: WWDRK     53   72483.1      |   7   19.690
Add: DENS      53   72494.9      |   7   19.700
Add: HUMID     53   72563.9      |   7   19.763
```

6) Some SAS output is shown below. Using the C(p) values from the output, find model $I_I = I_{min}$: give the (Number in model, R-square, C(p), Variables in Model).

Problem 6 Output

Number in Model	R-Square	C(p)	Variables in Model
1	0.1428	27.1534	x7
2	0.1635	22.1693	x3 x7
3	0.1907	14.9556	x4 x6 x7
4	0.2039	12.4831	x2 x4 x6 x7
5	0.2159	10.4328	x4 x6 x7 x9 x10
6	0.2316	7.0947	x3 x4 x6 x7 x9 x10
7	0.2374	7.1536	x2 x3 x4 x6 x7 x9 x10
8	0.2378	9.0000	x2 x3 x4 x5 x6 x7 x9 x10

X1 cause X2 age X3 height X4 headht X5 length X6 breadth X7 size X8 brainwt

X9 Circum X10 cephalic Sex X12 ageclass The circled terms are not included in the model.