Math 584 Quiz 2 Spring 2019 Name____________________

El rev 7, 8

1) Suppose that

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \sim N_4\left(\begin{pmatrix} 9 \\ 16 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 0.8 & -0.4 & 0 \\ 0.8 & 1 & -0.56 & 0 \\ -0.4 & -0.56 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right).$$

a) Find the distribution of X_3.

$N(4, 1)$

b) Find the distribution of $(X_2, X_4)^T$.

$N_2\left[\begin{pmatrix} 16 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]$

c) Which pairs of random variables X_i and X_j are independent?

$X_1 \perp\!\!\!\perp X_4$, $X_2 \perp\!\!\!\perp X_4$, $X_3 \perp\!\!\!\perp X_4$

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2) Recall that if $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \Sigma)$, then the conditional distribution of $\boldsymbol{X}_1$ given that $\boldsymbol{X}_2 = \boldsymbol{x}_2$ is multivariate normal with mean $\boldsymbol{\mu}_1 + \Sigma_{12}\Sigma_{22}^{-1}(\boldsymbol{x}_2 - \boldsymbol{\mu}_2)$ and covariance matrix $\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$. Let Y and X follow a bivariate normal distribution

El rev 9

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N_2\left(\begin{pmatrix} 15 \\ 20 \end{pmatrix}, \begin{pmatrix} 64 & 10 \\ 10 & 81 \end{pmatrix} \right).$$

a) Find $E(Y|X)$. $= 15 + \frac{10}{81}(x - 20) =$

$12.531 + \frac{10}{81}x$ $\boxed{= 12.531 + 0.1235x = \frac{1015}{81} + \frac{10}{81}x \approx 12.531 + .123x}$

b) Find $\mathrm{Var}(Y|X)$.

$= 64 - 10\frac{1}{81}10 = 64 - \frac{100}{81} = \boxed{62.7654}$

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2) Let $\boldsymbol{y} \sim N_3(\boldsymbol{\mu}, \sigma^2 \boldsymbol{I})$ where $\boldsymbol{y} = (Y_1, Y_2, Y_3)'$ and $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3)'$.

Let $\boldsymbol{A} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $\boldsymbol{B} = \frac{1}{6}\begin{bmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{bmatrix}$.

Are $\boldsymbol{y}'\boldsymbol{A}\boldsymbol{y}$ and $\boldsymbol{y}'\boldsymbol{B}\boldsymbol{y}$ independent? Explain.

craig's theorem $Y'AY \perp\!\!\!\perp Y'BY$ iff $A \Sigma B = 0$
if $B \Sigma A = 0$

so if $AB = 0$ or $BA = 0$

Since $AB = 0$ $\boxed{\text{Yes}}$

or

Th 2.5 $A\underline{Y} \perp\!\!\!\perp B\underline{Y}$ if $A \Sigma B' = 0$ so if

$A = A^2$ and $B = B^2$ then $Y'AY \perp\!\!\!\perp Y'BY$

so $\boxed{\text{Yes}}$

since then $\underline{Y}'A\underline{Y}$ is a fn of $A\underline{Y}$

$Y'B\underline{Y}$ $\quad$ $B\underline{Y}$

$$A^2 = \frac{1}{4}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = A$$

only needed to show $Y'AY \sim \chi^2_a$ $\quad Y'BY \sim \chi^2_b$

$$B^2 = \frac{1}{36}\begin{bmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{bmatrix} = \frac{1}{36}\begin{bmatrix} 6 & 6 & -12 \\ 6 & 6 & -12 \\ -12 & -12 & 24 \end{bmatrix} = B$$

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