Math 58~~8~~4 Quiz 5 Spring 2019 Name____________________

9 1) Suppose that $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i,4} + \epsilon_i$. Testing $H_0 : \beta_1 = 0$ is equivalent to testing $H_0 : \boldsymbol{A\beta} = \mathbf{0}$. What is $\boldsymbol{A}$?

Hint: Watch out for β_0.

$$\underset{1\times 5}{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad \underset{5\times 1}{\underline{\beta}} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}$$

10 2) Partition $\boldsymbol{X}$ as $\boldsymbol{X} = [\boldsymbol{X}_1 \ \ \boldsymbol{X}_2]$, let $\boldsymbol{P}$ be the projection matrix for $\mathcal{C}(\boldsymbol{X})$ and let $\boldsymbol{P}_1$ be the projection matrix for $\mathcal{C}(\boldsymbol{X}_1)$. Since $\mathcal{C}(\boldsymbol{P}_1) = \mathcal{C}(\boldsymbol{X}_1) \subseteq \mathcal{C}(\boldsymbol{X})$, $\boldsymbol{PP}_1 = \boldsymbol{P}_1$. Find $\boldsymbol{P}_1\boldsymbol{P}$. (Hint: consider $(\boldsymbol{PP}_1)'$.)

$$(PP_1)' = P_1' = P_1 = P_1'P' = P_1P$$

$$\text{so } P_1 P = \boxed{P_1} \qquad \left(= X_1'(X_1'X)^- X_1'\right)$$

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≈ 12.5 each

3) Let the full model be $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \epsilon_i$ and let the reduced model be $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i$ for $i = 1, ..., n$. Write the full model as $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\epsilon}$, and consider testing $H_0 : \boldsymbol{\beta}_2 = \boldsymbol{0}$. Let $\boldsymbol{P}_1$ be the projection matrix on $C(\boldsymbol{X}_1)$ and let $\boldsymbol{P}$ be the projection matrix on $C(\boldsymbol{X})$.

Then $F_R = \dfrac{n-p}{q}\dfrac{\boldsymbol{Y}'(\boldsymbol{P}-\boldsymbol{P}_1)\boldsymbol{Y}}{\boldsymbol{Y}'(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{Y}}$.

Assume $\boldsymbol{\epsilon} \sim N_n(\boldsymbol{0}, \sigma^2\boldsymbol{I})$. Assume H_0 is true.

a) What is q?

$5 - 3 = \boxed{2}$

or 2 predictors in full model not in reduced model

b) What is the distribution of $\boldsymbol{Y}'(\boldsymbol{P}-\boldsymbol{P}_1)\boldsymbol{Y}$? rank$(P-P_1) = tr(P-P_1) = q = 2$

$\dfrac{\underline{Y}'(P-P_1)\underline{Y}}{\sigma^2} \sim \chi^2_q$ so $\underline{Y}'(P-P_1)\underline{Y} \sim \sigma^2\chi^2_q = \boxed{\sigma^2\chi^2_2}$

c) What is the distribution of $\boldsymbol{Y}'(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{Y}$? rank$(I-P) = tr(I-P) = n-p = n-5$

$\dfrac{\underline{Y}'(I-P)\underline{Y}}{\sigma^2} \sim \chi^2_{n-p}$ so $\underline{Y}'(I-P)\underline{Y} \sim \sigma^2\chi^2_{n-p} = \boxed{\sigma^2\chi^2_{n-5}}$

d) What is the distribution of F_R? $F_{q,n-p} = \boxed{F_{2,n-5}}$

$$F_R = \frac{[\underline{Y}'(P-P_1)\underline{Y}/\sigma^2]/q}{[\underline{Y}'(I-P)\underline{Y}/\sigma^2]/(n-p)} = \frac{X_1/q}{X_2/(n-p)} \sim F_{q,n-p}$$

where $X_1 \sim \chi^2_q \perp\!\!\!\perp X_2 \sim \chi^2_{n-p}$

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