Math 588t Quiz 6 Fall 2014 spring 2004 Name____________________

1) Suppose that $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i,4} + \epsilon_i$. Testing $H_0 : \beta_1 = \beta_4 = 0$ is equivalent to testing $H_0 : \boldsymbol{A\beta} = \boldsymbol{0}$. What is $\boldsymbol{A}$?

Hint: Watch out for β_0.

$$\boxed{A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}} \qquad \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}$$

want $A\beta = \begin{pmatrix} \beta_1 \\ \beta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

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2) Let $\boldsymbol{Y} \sim N_n(\boldsymbol{X\beta}, \sigma^2 \boldsymbol{I}_n)$ where $\boldsymbol{X}$ is an $n \times p$ matrix of rank $r \leq p$. Let $\boldsymbol{X} = [\boldsymbol{X}_1\ \boldsymbol{X}_2]$ and $\boldsymbol{\beta} = (\boldsymbol{\beta}_1'\ \boldsymbol{\beta}_2')'$.

a) What is the distribution of $\boldsymbol{Y}'(\boldsymbol{I} - \boldsymbol{P})\boldsymbol{Y}$?

rank(P) = rank$(X) = r$

(E2 rev 46 v)

$$\frac{Y'(I-P)Y}{\sigma^2} \sim \chi^2_{tr(I-P)} \sim \chi^2_{n-r}$$

so $Y'(I-P)Y \sim \boxed{\sigma^2 \chi^2_{n-r}}$

b) Suppose that $H_0 : \boldsymbol{\beta}_2 = \boldsymbol{0}$ is true so that $\boldsymbol{Y} \sim N(\boldsymbol{X}_1\boldsymbol{\beta}_1, \sigma^2 \boldsymbol{I}_n)$. Let $\boldsymbol{P}_1$ be the projection matrix on $\mathcal{C}(\boldsymbol{X}_1)$. What is the distribution of $\boldsymbol{Y}'(\boldsymbol{I} - \boldsymbol{P}_1)\boldsymbol{Y}$ if $\boldsymbol{X}_1$ has full rank $k \leq r$?

rank(P_1) = rank$(X_1) = k$

$$\frac{Y'(I-P_1)Y}{\sigma^2} \sim \chi^2_{tr(I-P_1)} \sim \chi^2_{n-k}$$

so $Y'(I-P_1)Y \sim \boxed{\sigma^2 \chi^2_{n-k}}$

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3) Let the full model be $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + +\beta_6 x_{i6} + \beta_7 x_{i7} + \epsilon_i$ and let the reduced model be $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i$ for $i = 1, ..., n$. Write the full model as $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\epsilon}$, and consider testing $H_0 : \boldsymbol{\beta}_2 = \boldsymbol{0}$. Let $\boldsymbol{P}_1$ be the projection matrix on $C(\boldsymbol{X}_1)$ and let $\boldsymbol{P}$ be the projection matrix on $C(\boldsymbol{X})$.

Then $F_R = \dfrac{n-p}{q}\dfrac{\boldsymbol{Y}'(\boldsymbol{P}-\boldsymbol{P}_1)\boldsymbol{Y}}{\boldsymbol{Y}'(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{Y}}$.

Assume $\boldsymbol{\epsilon} \sim N_n(\boldsymbol{0}, \sigma^2\boldsymbol{I})$. Assume H_0 is true.

a) What is q?

$= 8 - 3 = \boxed{5} =$ # predictors in full model but not in reduced model

b) What is the distribution of $\boldsymbol{Y}'(\boldsymbol{P}-\boldsymbol{P}_1)\boldsymbol{Y}$?

$\dfrac{Y'(P-P_1)Y}{\sigma^2} \sim \chi^2_q$ so $\underline{Y}'(P-P_1)\underline{Y} \sim \sigma^2\chi^2_q \sim \boxed{\sigma^2\chi^2_5}$

c) What is the distribution of $\boldsymbol{Y}'(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{Y}$?

rank $(I-P) = n - p = n - 8$

$\dfrac{\underline{Y}'(I-P)\underline{Y}}{\sigma^2} \sim \chi^2_{n-p}$ so $\underline{Y}'(I-P)\underline{Y} \sim \sigma^2\chi^2_{n-p} \sim \boxed{\sigma^2\chi^2_{n-8}}$

d) What is the distribution of F_R?

$F_{q,n-p} \sim \boxed{F_{5,n-8}}$

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~1t each