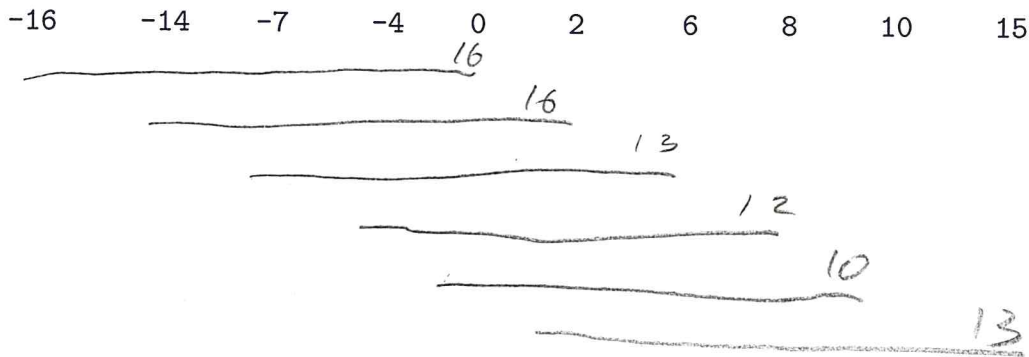


$(\beta_2 = \beta_3 = \beta_4 \text{ or } \beta_2 = \beta_3 \text{ is better})$
 $\beta_3 = \beta_4$

1) Suppose that $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \epsilon_i$. Testing $H_0 : \beta_2 = \beta_3 = \beta_4 = 0$ is equivalent to testing $H_0 : A\beta = 0$. What is A ?

$A\beta = A \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} = \begin{pmatrix} \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$ so $A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

2) Find shorth(5) for the following data set of ordered residuals. Show work.



Shorth(5) = [0, 10]

3) Find the vector a such that $a^T Y$ is an unbiased estimator for $E(Y_i)$ if the usual linear model holds.

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$a = (0, \dots, 0, 1, 0, \dots, 0)^T$

↑
i-th place

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then $a^T Y = Y_i$ 1 $Y = (Y_1, \dots, Y_i, Y_{i+1}, \dots, Y_n)^T$

$$E(Y) = X\beta = \begin{pmatrix} \beta_1 - \beta_2 \\ \beta_1 - \beta_2 \end{pmatrix} \Rightarrow X = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \Rightarrow X^T = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$C(X^T) = \text{span} \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

4) Suppose that $Y = (Y_1, Y_2)'$, $\text{Var}(Y) = \sigma^2 I$, $E(Y_1) = E(Y_2) = \beta_1 - \beta_2$. Show whether or not the following functions are estimable. Hint $E(Y) = X\beta$, so find X .

a) $\beta_1 = (1 \ 0) \beta$, $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \notin C(X^T)$

NO

b) $\beta_2 = (0 \ 1) \beta$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \notin C(X^T)$

NO

c) $\beta_1 - \beta_2 = (1 \ -1) \beta$, $\begin{pmatrix} 1 \\ -1 \end{pmatrix} \in C(X^T)$

YES

(also if $c = (1 \ 0)'$, $E(c'Y) = E(Y_1) = \beta_1 - \beta_2$, $d = (0 \ 1)'$ works too)

d) $\beta_1 + \beta_2 = (1 \ 1) \beta$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \notin C(X^T)$

NO