($B_2 = B_3 = B_4$ or $B_2 = B_3$, $B_3 = B_4$ is better)

Math 584 Quiz 7 Spring 2019 Name____________________

1) Suppose that $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i,4} + \beta_5 x_{i,5} + \epsilon_i$. Testing $H_0 : \beta_2 = \beta_3 = \beta_4 = 0$ is equivalent to testing $H_0 : \boldsymbol{A\beta} = \boldsymbol{0}$. What is $\boldsymbol{A}$?

$$A\beta = A \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} = \begin{pmatrix} \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} \text{ so } A = \boxed{\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}$$

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2) Find shorth(5) for the following data set of ordered residuals. Show work.

-16 -14 -7 -4 0 2 6 8 10 15

16

16

13

12

10

13

$\boxed{\text{shorth}(5) = [0, 10]}$

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3) Find the vector $\boldsymbol{a}$ such that $\boldsymbol{a}^T\boldsymbol{Y}$ is an unbiased estimator for $E(Y_i)$ if the usual linear model holds.

$$\boxed{\underset{\sim}{a} = (0, \ldots, 0, 1, 0, \ldots, 0)^T}$$

ith place

then $\underline{a}'\underline{Y} = Y_i$, $\underline{Y} = (Y_1, \ldots, Y_i, Y_{i+1}, \ldots, Y_n)'$

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$E(\underline{Y}) = \underline{X}\,\underline{\beta} = \begin{pmatrix} \beta_1-\beta_2 \\ \beta_1-\beta_2 \end{pmatrix} \Rightarrow \underline{X} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \Rightarrow \underline{X}^T = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$C(\underline{X}^T) = \text{span}\left\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$

4) Suppose that $\boldsymbol{Y} = (Y_1, Y_2)'$, $\text{Var}(\boldsymbol{Y}) = \sigma^2\boldsymbol{I}$, $E(Y_1) = E(Y_2) = \beta_1 - \beta_2$. Show whether or not the following functions are estimable. Hint $E(\boldsymbol{Y}) = \boldsymbol{X\beta}$, so find $\boldsymbol{X}$.

a) β_1 $= (1 \;\; 0)\underline{\beta}$, $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \notin C(\underline{X}^T)$

no

b) β_2 $= (0 \;\; 1)\underline{\beta}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \notin C(\underline{X}^T)$

no

c) $\beta_1 - \beta_2$ $= (1 \;\; -1)\underline{\beta}$, $\begin{pmatrix} 1 \\ -1 \end{pmatrix} \in C(\underline{X}^T)$

Yes

(also if $\underline{c} = (1 \;\; 0)'$, $E(\underline{c}'\underline{Y}) = E(Y_1) = \beta_1 - \beta_2$, $\underline{d} = (0 \;\; 1)'$ works too)

d) $\beta_1 + \beta_2$ $(1 \;\; 1)\underline{\beta}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \notin C(\underline{X}^T)$

no

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$\underline{a}^T\underline{\beta}$ is estimable iff $\underline{a} \in C(\underline{X}^T)$ if $\exists\, \underline{c}^T\underline{Y} \ni E(\underline{c}^T\underline{Y}) = \underline{a}'\underline{\beta}$