Math 584 Quiz 8 Fall 2019 Name____________________

Current terms: (EDUC JANT log[NONW] log[NOX] OVR65 PREC)

	df	RSS	|	k	C_I
Delete: OVR65	54	59897.9	|	6	6.284
Delete: EDUC	54	66809.3	|	6	12.547
Delete: log[NONW]	54	73178.1	|	6	18.319
Delete: JANT	54	76417.1	|	6	21.255
Delete: PREC	54	83958.1	|	6	28.089
Delete: log[NOX]	54	86823.1	|	6	30.685

1) From the above output, which model should be used?

6 terms with constant

$\boxed{\text{constant, EDUC, JANT, log(NONW), log(NOX), PREC}}$

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Base terms: (x3)

	df	RSS	|	k	C_I
Add: x6	109	43441.6	|	3	0.793
Add: x4	109	48569.8	|	3	13.399
Add: x5	109	49618.9	|	3	15.978
Add: x2	109	49772.8	|	3	16.356

e

2) The above output is for forward selection. The full model has a constant x_1 and predictors $x_2 =$ *height while sitting*, $x_3 =$ *height while kneeling*, $x_4 =$ *head length*, $x_5 =$ *nasal breadth*, and $x_6 =$*arm span*. Hence the full model has 6 terms. What is the model I_{min} that has the smallest $C_p(I)$?

$\boxed{\text{constant, x3, x6}}$

$\underbrace{\text{constant}}_{= x1}$

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3) Suppose $\boldsymbol{Y}^* \sim N_n(\boldsymbol{X}\hat{\boldsymbol{\beta}}, \sigma_n^2 \boldsymbol{I}_n)$. Hence $Y_i^* = \boldsymbol{x}_i^T\hat{\boldsymbol{\beta}} + \epsilon_i^P$ where $E(\epsilon_i^P) = 0$ and $V(\epsilon_i^P) = \sigma_n^2$. Hence $\boldsymbol{A}\boldsymbol{Y}^* \sim N_g(\boldsymbol{A}\boldsymbol{X}\hat{\boldsymbol{\beta}}, \sigma_n^2 \boldsymbol{A}\boldsymbol{A}^T)$ if $\boldsymbol{A}$ is a $g \times n$ constant matrix. Recall that $\boldsymbol{X}$ is an $n \times p$ constant matrix. Simplify quantities when possible.

20 a) What is the distribution of $\hat{\boldsymbol{\beta}}^* = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y}^*$?

$$N_p\left((X'X)^{-1}X'X\underline{\hat{\beta}},\ \sigma_n^2 (X'X)^{-1}X'X(X'X)^{-1}\right)$$

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$$\sim N_p\left[\underline{\hat{\beta}},\ \sigma_n^2 (X'X)^{-1}\right]$$

20 b) Using a), what is $E(\hat{\boldsymbol{\beta}}^*)$?

$$\boxed{\underline{\hat{\beta}}}$$

10 c) Recall that $\boldsymbol{X}\hat{\boldsymbol{\beta}} = \boldsymbol{P}\boldsymbol{Y}$. What is the distribution of $\hat{\boldsymbol{\beta}}_I^* = (\boldsymbol{X}_I^T\boldsymbol{X}_I)^{-1}\boldsymbol{X}_I^T\boldsymbol{Y}^*$?

$k \times 1$

$$N_k\left[\underbrace{(X_I'X_I)^{-1}\underbrace{X_I'PY}_{X_I'}}_{\hat{\beta}_I},\ \sigma_n^2 (X_I'X_I)^{-1}X_I'X_I(X_I'X_I)^{-1}\right]$$

$$\sim \boxed{N_k\left[\underline{\hat{\beta}}_I,\ \sigma_n^2 (X_I'X_I)^{-1}\right]}$$

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