Math 584 Quiz 2 Spring 2021   Name____________________

1) Suppose that

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \sim N_4\left(\begin{pmatrix} 49 \\ 25 \\ 9 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 & -1 & 3 & 0 \\ -1 & 5 & -3 & 0 \\ 3 & -3 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \right).$$

a) Find the distribution of X_2.

b) Find the distribution of $(X_1, X_3)^T$.

c) Find the correlation $\rho(X_1, X_3)$.

2) Recall that if $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then the conditional distribution of $\boldsymbol{X}_1$ given that $\boldsymbol{X}_2 = \boldsymbol{x}_2$ is multivariate normal with mean $\boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\boldsymbol{x}_2 - \boldsymbol{\mu}_2)$ and covariance matrix $\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$. Let Y and X follow a bivariate normal distribution

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N_2\left(\begin{pmatrix} 49 \\ 17 \end{pmatrix}, \begin{pmatrix} 3 & -1 \\ -1 & 4 \end{pmatrix} \right).$$

a) Find $E(Y|X)$.

b) Find $\text{Var}(Y|X)$.

3) Let $\boldsymbol{y} \sim N_2(\boldsymbol{\mu}, \sigma^2 \boldsymbol{I})$ where $\boldsymbol{y} = (Y_1, Y_2)'$ and $\boldsymbol{\mu} = (\mu_1, \mu_2)'$. Let $\boldsymbol{A} = \left[\begin{array}{cc} 1/2 & 1/2 \\ 1/2 & 1/2 \end{array} \right]$ and $\boldsymbol{B} = \left[\begin{array}{cc} 1/2 & -1/2 \\ -1/2 & 1/2 \end{array} \right]$.

a) Are $\boldsymbol{Ay}$ and $\boldsymbol{By}$ independent? Explain. Hint: The quantities are independent if $Cov(\boldsymbol{Ay}, \boldsymbol{By}) = \boldsymbol{0}$.

b) Are $\boldsymbol{y}'\boldsymbol{Ay}$ and $\boldsymbol{y}'\boldsymbol{By}$ independent? Explain.