Math 584 Quiz 4 Fall 2021 Name____________________

1) Suppose $Y_i = \boldsymbol{x}_i^T \boldsymbol{\beta} + \epsilon_i$ with $Q(\boldsymbol{\beta}) \geq 0$. Let c_n be a constant that does not depend on $\boldsymbol{\beta}$ or σ^2. Suppose the likelihood function is

$$L(\boldsymbol{\beta}, \sigma^2) = c_n \, \frac{1}{\sigma^n} \exp\left(\frac{-1}{2\sigma^2} Q(\boldsymbol{\beta}) \right).$$

a) Suppose that $\hat{\boldsymbol{\beta}}_Q$ minimizes $Q(\boldsymbol{\beta})$. By direct maximization, show that $\hat{\boldsymbol{\beta}}_Q$ is the MLE of $\boldsymbol{\beta}$ regardless of the value of σ.

b) Then find the MLE $\hat{\tau} = \hat{\sigma}^2$ of $\sigma^2 = \tau$ by maximizing

$$L_P(\tau) \equiv L(\hat{\boldsymbol{\beta}}_Q, \tau) = c_n \frac{1}{\tau^{n/2}} \exp(\frac{-1}{2\tau} Q(\hat{\boldsymbol{\beta}})).$$

2) By the least squares central limit theorem, $\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \xrightarrow{D} N_p(\mathbf{0}, \sigma^2\ \boldsymbol{W})$. Hence the limiting distribution of of $\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$ is the $N_p(\mathbf{0}, \sigma^2\ \boldsymbol{W})$ distribution. Let $\boldsymbol{A}$ be a constant $r \times p$ matrix. Find the limiting distribution of $\boldsymbol{A}\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$.

3) Suppose $\boldsymbol{Z}_n \xrightarrow{D} N_k(\boldsymbol{\mu}, \boldsymbol{I})$. Let $\boldsymbol{A}$ be a constant $r \times k$ matrix. Find the limiting distribution of $\boldsymbol{A}(\boldsymbol{Z}_n - \boldsymbol{\mu})$.