Math 583 Quiz 5 Fall 2021 Name____________________

1) Suppose that $Y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_{p-1} x_{i,p-1} + \epsilon_i$. Let $\boldsymbol{\beta} = (\boldsymbol{\beta}_1' \ \boldsymbol{\beta}_2')'$ where $\boldsymbol{\beta}_1 = (\beta_0, ..., \beta_{p-q-1})'$ is a $(p-q) \times 1$ vector and $\boldsymbol{\beta}_2 = (\beta_{p-q}, ..., \beta_{p-1})'$ is a $q \times 1$ vector where $q \geq 1$. Consider testing $H_0 : \boldsymbol{\beta}_2 = \boldsymbol{0}$. Cases with $q = p-1$ and $q = 1$ are especially interesting.

a) Testing $H_0 : \boldsymbol{\beta}_2 = \boldsymbol{0}$ is equivalent to testing $H_0 : \boldsymbol{A\beta} = \boldsymbol{0}$. What is $\boldsymbol{A}$?

b) Writing $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon} = [\boldsymbol{X}_1 \ \boldsymbol{X}_2](\boldsymbol{\beta}_1' \ \boldsymbol{\beta}_2')' + \boldsymbol{\epsilon} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\epsilon}$, if $H : \boldsymbol{\beta}_2 = 0$ is true, then $\boldsymbol{Y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{\epsilon}$. Assume that $\boldsymbol{X}$ is an $n \times p$ matrix of rank p. What is the projection matrix $\boldsymbol{P}_{\boldsymbol{X}_1}$ on $C(\boldsymbol{X}_1)$? (Make $\boldsymbol{P}_{\boldsymbol{X}_1}$ as simple as possible.)

2) Let $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon}$. Is $\boldsymbol{P}_{\boldsymbol{X}}\boldsymbol{Y} = \boldsymbol{Y}$?

3) Let the full model be $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \epsilon_i$ and let the reduced model be $Y_i = \beta_0 + \beta_2 x_{i2} + \epsilon_i$ for $i = 1, ..., n$. Write the full model as $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\epsilon}$, and consider testing $H_0 : \boldsymbol{\beta}_2 = \boldsymbol{0}$ where $\boldsymbol{\beta}_1$ corresponds to the reduced model. Let $\boldsymbol{P}_1$ be the projection matrix on $C(\boldsymbol{X}_1)$ and let $\boldsymbol{P}$ be the projection matrix on $C(\boldsymbol{X})$.

Then $F_R = \dfrac{n-p}{q} \dfrac{\boldsymbol{Y}'(\boldsymbol{P} - \boldsymbol{P}_1)\boldsymbol{Y}}{\boldsymbol{Y}'(\boldsymbol{I} - \boldsymbol{P})\boldsymbol{Y}}$.

Assume $\boldsymbol{\epsilon} \sim N_n(\boldsymbol{0}, \sigma^2 \boldsymbol{I})$. Assume H_0 is true.

a) What is q?

b) What is the distribution of $\boldsymbol{Y}'(\boldsymbol{P} - \boldsymbol{P}_1)\boldsymbol{Y}$?

c) What is the distribution of $\boldsymbol{Y}'(\boldsymbol{I} - \boldsymbol{P})\boldsymbol{Y}$?

d) What is the distribution of F_R?