Math 584 Quiz 6 Spring 2021 Name____________________

1) Suppose that $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i,4} + \epsilon_i$. Testing $H_0 : \beta_1 = \beta_3 = \beta_4 = 0$ is equivalent to testing $H_0 : \boldsymbol{A\beta} = \boldsymbol{0}$. What is $\boldsymbol{A}$? Hint: Watch out for β_0.

2) Let $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon}$ where $E(\boldsymbol{\epsilon}) = \boldsymbol{0}$, $Cov(\boldsymbol{\epsilon}) = \sigma^2 \boldsymbol{I}_n$, and $\boldsymbol{X}$ has full rank. Note that $Y_i = \boldsymbol{x}_i^T \boldsymbol{\beta} + \epsilon_i$. Assume $\boldsymbol{X}$ is a constant matrix.

a) Find $E(Y_i)$.

b) Is $E(Y_i)$ estimable? Explain briefly. Hint: Recall that $\boldsymbol{a}'\boldsymbol{\beta}$ is estimable if $\boldsymbol{a} \in C(\boldsymbol{X}')$ or if $E(\boldsymbol{b}'\boldsymbol{Y}) = \boldsymbol{a\beta}$. For the full rank model, $\boldsymbol{a}'\boldsymbol{\beta}$ is estimable by $\boldsymbol{a}'\hat{\boldsymbol{\beta}}$. Here $\boldsymbol{a}$ and $\boldsymbol{b}$ are constant vectors.

3) Let the full model be $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \beta_6 x_{i6} + \beta_7 x_{i7} + \epsilon_i$ and let the reduced model be $Y_i = \beta_0 + \beta_1 x_{i1} + \epsilon_i$ for $i = 1, ..., n = 103$. Write the full model as $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\epsilon}$, and consider testing $H_0 : \boldsymbol{\beta}_2 = \boldsymbol{0}$. Let $\boldsymbol{P}_1$ be the projection matrix on $C(\boldsymbol{X}_1)$ and let $\boldsymbol{P}$ be the projection matrix on $C(\boldsymbol{X})$.

Then $F_R = \dfrac{n-p}{q} \dfrac{\boldsymbol{Y}'(\boldsymbol{P} - \boldsymbol{P}_1)\boldsymbol{Y}}{\boldsymbol{Y}'(\boldsymbol{I} - \boldsymbol{P})\boldsymbol{Y}}$.

Assume $\boldsymbol{\epsilon} \sim N_n(\boldsymbol{0}, \sigma^2\boldsymbol{I})$. Assume H_0 is true.

a) What is q?

b) What is the distribution of $\boldsymbol{Y}'(\boldsymbol{P} - \boldsymbol{P}_1)\boldsymbol{Y}$?

c) What is the distribution of $\boldsymbol{Y}'(\boldsymbol{I} - \boldsymbol{P})\boldsymbol{Y}$?

d) What is the distribution of F_R?