Math 584 Quiz 8 Spring 2021 Name____________________

k	CP	ADJUSTED R SQUARE	R SQUARE	RESID SS	MODEL VARIABLES
			97 cases after deleting the 2 outliers		
1	903.5	0.0000	0.0000	183.102	INTERCEPT ONLY
2	0.7	0.9052	0.9062	17.1785	B
2	406.6	0.4944	0.4996	91.6174	A
2	426.0	0.4748	0.4802	95.1708	C
3	2.1	0.9048	0.9068	17.0741	A C
3	2.6	0.9043	0.9063	17.1654	B C
3	2.6	0.9042	0.9062	17.1678	A B
4	4.0	0.9039	0.9069	17.0539	A B C

1) The output above is from software that does all subsets variable selection. The data is from Ashworth (1842). The predictors were A = log(1692 property value), B = log(1841 property value) and C = log(percent increase in value) while the response variable is Y = log(1841 population). All models contain an intercept=constant. Find model I_I that has the fewest number of predictors such that $C_p(I_I) \leq C_p(I_{min}) + 1$. This model is the initial model to examine.

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Base terms: Intercept
                df   RSS          |  k    C_I
Add: log[D]     29  0.383244      |  2   30.860
Add: log[Ht]    29   4.81297      |  2  699.629
Base terms: (log[D])
                df   RSS          |  k    C_I
Add: log[Ht]
                28  0.185463      |  3    3.000
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2) The above output is for 31 black cherry trees. D = diameter of the tree 4.5 feet of the ground in inches, Ht = height of the tree in feet, Vol = marketable volume of wood in cubic feet. $Y = log(Vol)$ is the response variable while log(D) and log(Ht) are possible predictors. All models have a constant. From the above output, what model should be used? Do not forget the constant.

3) Suppose $\boldsymbol{Y}^* = \boldsymbol{X}\hat{\boldsymbol{\beta}} + \boldsymbol{r}^W$ where where $E(\boldsymbol{r}^W) = \boldsymbol{0}$ and $Cov(\boldsymbol{r}^W) = Cov(\boldsymbol{Y}^*) = MSE\ \boldsymbol{I}_n$. Then $\hat{\boldsymbol{\beta}}^* = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y}^*$. Recall that $\boldsymbol{X}$ is an $n \times p$ constant matrix. Simplify quantities when possible.

a) What is $E(\hat{\boldsymbol{\beta}}^*)$?

b) What is $Cov(\hat{\boldsymbol{\beta}}^*)$?

c) Recall that $\boldsymbol{X}\hat{\boldsymbol{\beta}} = \boldsymbol{P}\boldsymbol{Y}$. What is $E(\hat{\boldsymbol{\beta}}^*_I) = E[(\boldsymbol{X}_I^T\boldsymbol{X}_I)^{-1}\boldsymbol{X}_I^T\boldsymbol{Y}^*]$?

d) What is $Cov(\hat{\boldsymbol{\beta}}^*_I)$?