Math 585, HW 2 Spring 2024, Due Friday, Feb. 2. Two pages, problems A)-E).

A) This problem makes plots similar to Figure 2.1. Data sets of $n = 100$ cases from two $N_2(\mathbf{0}, \boldsymbol{\Sigma}_i)$ distributions are generated and plotted in a scatterplot along with the 10%, 30%, 50%, 70%, 90% and 98% highest density regions where

$$\boldsymbol{\Sigma}_1 = \begin{pmatrix} 1 & 0.9 \\ 0.9 & 4 \end{pmatrix} \text{ and } \boldsymbol{\Sigma}_2 = \begin{pmatrix} 1 & -0.4 \\ -0.4 & 1 \end{pmatrix}.$$

On the computer, double click on the *Arc* icon. (Using the mouse, move the pointer (cursor) to the icon and press the leftmost mouse button twice, rapidly. This procedure is known as *double clicking* on the icon.) The *Arc* window should appear with a "greater than" > prompt. The menu *File* should be in the upper left corner of the window. Move the pointer to *File* and hold the leftmost mouse button down. Then the menu will appear. Drag the pointer down to the menu command *load.* Then click on *data* and then click on *demo-bn.lsp.* (You may need to use the *slider bar* in the middle of the screen to see the file *demo-bn.lsp*: click on the arrow pointing to the right until the file appears.) In the future these menu commands will be denoted by "File > Load > Data > demo-bn.lsp." These are the commands needed to activate the file *demo-bn.lsp.*

a) In the *Arc* dialog window, enter the numbers
0 0 1 4 0.9 and 100. Then click on *OK.*

The graph can be printed with the menu commands "File>Print," but it will generally save paper by placing the plots in the *Word* editor.

Activate *Word* (often by double clicking on the *start icon* and then the *Word* icon). Click on the screen and type "Problem 2Aa." In *Arc,* use the menu commands "Edit>Copy." In *Word,* click on the *Paste* icon near the upper left corner of *Word* and hold down the leftmost mouse button. This will cause a menu to appear. Drag the pointer down to *Paste.* The plot should appear on the screen. (Older versions of *Word,* use the menu commands "Edit>Paste.") **In the future**, "paste the output into *Word*" will refer to these mouse commands.

b) Either click on *new graph* on the current plot in *Arc* or reload *demo-bn.lsp.* In the *Arc* dialog window, enter the numbers
0 0 1 1 -0.4 and 100. Then place the plot in *Word.*

After editing your *Word* document, get a printout by clicking on the upper left *icon*, select "Print" then select "Print". (Older versions of *Word* use the menu commands "File>Print.")

To save your output on your flash drive J, click on the icon in the upper left corner of *Word.* Then drag the pointer to "Save as." A window will appear, click on the *Word Document* icon. A "Save as" screen appears. Click on the right "check" on the top bar, and then click on "Removable Disk (J:)". Change the file name to HW2A.docx, and then click on "Save."

To exit from *Word* and *Arc*, click on the "X" in the upper right corner of the screen. In *Word* a screen will appear and ask whether you want to save changes made in your document. Click on *No.* In *Arc,* click on *OK.*

Arc is described in more detail in Section 15.2 and Cook and Weisberg (1999a).

B), 3.15 Suppose that

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \sim N_4\left(\begin{pmatrix} 9 \\ 16 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 0.8 & -0.4 & 0 \\ 0.8 & 1 & -0.56 & 0 \\ -0.4 & -0.56 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right).$$

a) Find the distribution of X_3.

b) Find the distribution of $(X_2, X_4)^T$.

c) Which pairs of random variables X_i and X_j are independent?

d) Find the correlation $\rho(X_1, X_3)$.

C) 3.3. Recall that if $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then the conditional distribution of $\boldsymbol{X}_1$ given that $\boldsymbol{X}_2 = \boldsymbol{x}_2$ is multivariate normal with mean $\boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\boldsymbol{x}_2 - \boldsymbol{\mu}_2)$ and covariance $\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}$.

Let $\sigma_{12} = \text{Cov(Y,X)}$ and suppose Y and X follow a bivariate normal distribution

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N_2\left(\begin{pmatrix} 15 \\ 20 \end{pmatrix}, \begin{pmatrix} 64 & \sigma_{12} \\ \sigma_{12} & 81 \end{pmatrix} \right).$$

a) If $\sigma_{12} = 10$ find $E(Y|X)$.
b) If $\sigma_{12} = 10$, find $\text{V}(Y|X)$.
c) If $\sigma_{12} = 10$, find $\rho(Y, X)$, the correlation between Y and X.
d) What is σ_{12} if Y and X are independent?

D) 3.4. Suppose that

$$\boldsymbol{X} \sim (1-\gamma)EC_p(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g_1) + \gamma EC_p(\boldsymbol{\mu}, c\boldsymbol{\Sigma}, g_2)$$

where $c > 0$ and $0 < \gamma < 1$. Following Example 3.2, show that $\boldsymbol{X}$ has an elliptically contoured distribution assuming that all relevant expectations exist.

E) Let $\boldsymbol{X}$ be an $n \times p$ constant matrix and let $\boldsymbol{\beta}$ be a $p \times 1$ constant vector. Suppose $\boldsymbol{Y} \sim N_n(\boldsymbol{X\beta}, \sigma^2\boldsymbol{I})$. Find the distribution of $(\boldsymbol{I}-\boldsymbol{H})\boldsymbol{Y}$ if $(\boldsymbol{I}-\boldsymbol{H})^T = (\boldsymbol{I}-\boldsymbol{H}) = (\boldsymbol{I}-\boldsymbol{H})^2$ is an $n \times n$ matrix and if $\boldsymbol{HX} = \boldsymbol{X}$. Simplify.