

1) Suppose  $\mathbf{x}_1, \dots, \mathbf{x}_n$  are iid  $2 \times 1$  random vectors from a multivariate lognormal  $LN(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  distribution. Let  $\mathbf{x}_i = (X_{i1}, X_{i2})^T$ . Following Press (2005, p. 149-150),  $E(X_{ij}) = \exp(\mu_j + \sigma_j^2/2)$ ,  $V(X_{ij}) = \exp(\sigma_j^2)[\exp(\sigma_j^2) - 1] \exp(2\mu_j)$  for  $j = 1, 2$ , and  $\text{Cov}(X_{i1}, X_{i2}) = \exp[\mu_1 + \mu_2 + 0.5(\sigma_1^2 + \sigma_2^2) + \sigma_{12}][\exp(\sigma_{12}) - 1]$ . Find the limiting distribution of  $\sqrt{n}(\bar{\mathbf{x}} - \mathbf{c})$  for appropriate vector  $\mathbf{c}$ .

$$\sqrt{n} \left[ \bar{\mathbf{x}} - \begin{pmatrix} \exp(\mu_1 + \frac{\sigma_1^2}{2}) \\ \exp(\mu_2 + \frac{\sigma_2^2}{2}) \end{pmatrix} \right] \xrightarrow{D}$$

$$N_2 \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \exp(\sigma_1^2)[\exp(\sigma_1^2)-1]\exp(2\mu_1) & \text{Cov}(X_{i1}, X_{i2}) \\ \text{Cov}(X_{i1}, X_{i2}) & \exp(\sigma_2^2)[\exp(\sigma_2^2)-1]\exp(2\mu_2) \end{pmatrix} \right)$$

$$\begin{pmatrix} V(X_{i1}) & \text{Cov}(X_{i1}, X_{i2}) \\ \text{Cov}(X_{i1}, X_{i2}) & V(X_{i2}) \end{pmatrix}$$

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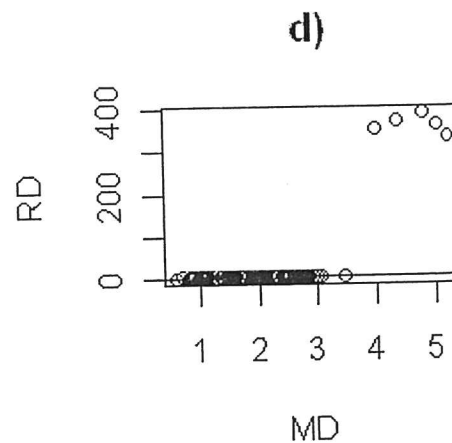
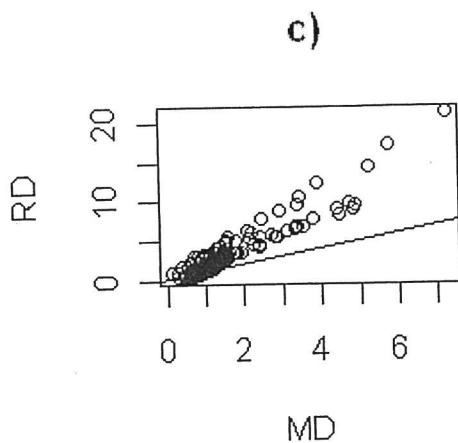
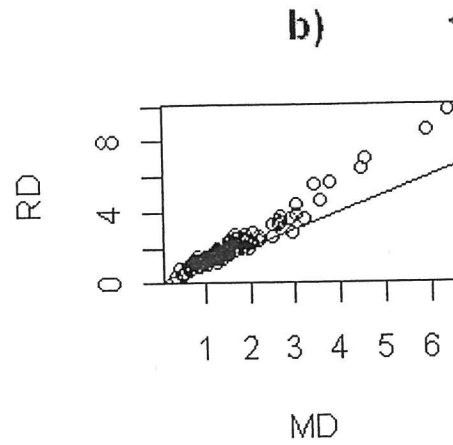
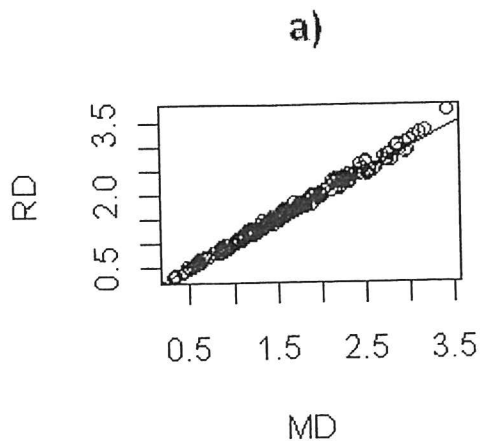
2) The table  $W$  shown below represents 5 measurements on 5 people. Find the sample mean  $\bar{\mathbf{x}}$  and the coordinatewise median  $\text{MED}(W)$ .

	len	nasal	bigonal	cephalic	height	MED(W)				
	174	178	181	184	1755					
	46	53	54	55	59					
	100	102	103	106	110					
	75.69	79.78	83.33	83.7	86.63					
	18	1670	1697	1723	1755					
Sums	2472	267	521	409.13	6863					

} order each variable  
} height

$$\bar{\mathbf{x}} = \frac{1}{5} \begin{pmatrix} 2472 \\ 267 \\ 521 \\ 409.13 \\ 6863 \end{pmatrix} = \begin{pmatrix} 494.4 \\ 53.4 \\ 104.2 \\ 81.826 \\ 1372.6 \end{pmatrix}$$

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3) Shown above are 4 DD plots. Classify the data distribution as multivariate normal, elliptically contoured but not multivariate normal, not elliptically contoured or outliers are present. Explain your choices briefly.

- a) MVN      points follow identity line
- b) EC not MVN      points follow line with slope  $> 1$
- c) not EC      (or outliers) data does not follow a line through the origin
- d) outliers      since some points are far from the bulk of the data

1) c) 4 points - 17  
 1) w/ 100 - 17  
 2) w/ 100 - 20