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9)

$$pval = P\left(\frac{-[n-p+1-0.5(m-r+3)]}{rm} \log(\Lambda(L)) < F_{rm,n-rm}\right)$$

$$pval = P\left(\frac{n-p}{rm} V(L) < F_{rm,n-rm}\right)$$

$$pval = P\left(\frac{n-p}{rm} U(L) < F_{rm,n-rm}\right)$$

$$pval \approx P\left(\frac{n-p-h+r}{h} \lambda_{max}(L) < F_{h,n-p-h+r}\right)$$

10) The MANOVA F test uses

$$\underset{(p-1)\times p}{L} = [\underline{0} \; I_{p-1}]$$

i) H_0 the nontrivial predictors are not needed in the mreg model

H_1 at least one of the nontrivial predictors is needed.

ii) Get F_0 from output.

iii) Get pval from output.

iv) If pval $< \alpha$ reject H_0, there is a mreg relationship between the response variables $Y_1, ..., Y_m$ and the predictors $x_2, ..., x_p$. If pval $\geq \alpha$

fail to reject H_o, there is not a mreg relationship between $Y_1, \ldots, Y_m$ and $x_2, \ldots, x_p$. (Get variable names from the story problem.)

usually have a constant in the full and reduced models,

11) The MANOVA partial F test has a full model using all of the predictors and a reduced model where $r \geq 1$ of the predictors are deleted. The ith row of L has 0s except for a 1 in the position corresponding to the ith variable to be deleted.

i) H_o the reduced model is good H_1 use the full model

ii) Get F_R from output.

iii) Get pval from output.

iv) If pval $< \alpha$, reject H_o, conclude the full model should be used.

If pval $\geq \alpha$, fail to reject H_o, conclude the reduced model is good.

12) The F_j test uses $L_j = [0, \ldots, 0, \overset{j\text{th position}}{1}, 0, \ldots, 0]$

i) H_o $\underline{b}_j^T = \underline{0}^T$ $\quad H_1$ $\underline{b}_j^T \neq \underline{0}^T$

ii) Get F_j from output.

iii) Get pval from output.

iv) If pval $< \alpha$, reject H_o, conclude x_j is needed in the mreg model. If pval $\geq \alpha$, fail to reject H_o. conclude x_j is not needed in the mreg model given the other predictors are in the model.

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13) The MANOVA F test should reject Ho if n is large, the response and residual plots look good, and at least one response plot does not look like the corresponding residual plot. [The response plot looks like a residual plot if the identity line appears almost horizontal, so the range of $\hat{Y}_j$ is small.)

14) The mpack function mltreg produces the m response and residual plots, $\hat{B}$, $\hat{\Sigma}_{\epsilon}$, output for the MANOVA partial F test corresponding to variables given by indices being left out of the full model, output for the F_j tests for $j=1$ (constant), 2, ..., p, and output for the MANOVA F test. See E3 rev 128). Note: indices = c(j) will have a MANOVA partial F test corresponding to the F_j test while indices = c(2, ..., p) will have a MANOVA partial F test corresponding to the MANOVA F test. So the MANOVA F test and F_j tests are special cases of the MANOVA partial F test. Hotelling Lawley is used,

ex) see $\underline{\text{E3rev 128)}}$ (output), HW11c, Fitness data

Y_1 = chinups, Y_2 = situps, Y_3 = jumps

x_2 = weight, X_3 = waist, X_4 = pulse

a) MANOVA F test

i) Ho the nontrivial predictors are not needed in the mreg model
H_1 at least one of the nontrivial predictors is needed

ii) $F_0 = 3.150$

iii) pval = .060

iv) fail to reject Ho, there is not a mreg relationship between chinups situps, jumps and the predictors weight, waist, and pulse.

Note reject Ho if $\alpha = .061$ or $\alpha = 0.1$.

b) F_2 test

i) Ho $\underline{b}_2^T = \underline{0}$ $\quad H_1$ $\underline{b}_2^T \neq \underline{0}$

ii) $F_2 = 1.510$

iii) pval = .284

iv) fail to reject Ho, x_2 = weight is not needed in the mreg model given the other predictors are in the model.

c) F3 test

i) Ho $\underline{b}_3^T = \underline{0}$ H_1 $\underline{b}_3^T \neq \underline{0}$

ii) $F_3 = 5.613$

iii) pval = .022

iv) reject Ho x_3 = waist is needed in the mreg model

d) test whether the reduced model that deletes x_2 and x_4 is good

i) Ho the reduced model is good H_1 use the full model

ii) $F_R = 0.770$

iii) Pval = 0.614

iv) fail to reject Ho, the reducedmodel is good

e) test whether the reduced model that deletes x_3 and x_4 is good if

partial F	Pval
2.887	.084

i) Ho the reduced model is good H_1 use the full model

ii) $F_R = 2.887$

iii) pval = 0.084

iv) fail to reject Ho, the reduced model is good (reject Ho if $\alpha = 0.1$)

49.9

15) There is a robust estimator that replaces least squares by hbreg.

The probability that the hbreg estimator = the least squares estimator goes to 1 as $n \to \infty$ for a large class of EC iid errors $\underset{\sim}{\epsilon}_i$. Hence the large sample tests and prediction regions for the classical multivariate linear regression estimator are also large sample tests and prediction regions for the robust multivariate regression estimator.

Need n pretty large before the inference performs well.

16) The rmreg2 estimator is better, and replaces covariance matrix estimators by robust estimators.

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17) Prediction region for $\underset{\sim}{y}_f$ given predictors $\underset{\sim}{x}_f$. $\underset{\sim}{z} \sim \underset{\sim}{y}_f | \underset{\sim}{x}_f$ follows a multivariate distribution with mean $E(\underset{\sim}{z}) = E(\underset{\sim}{y}_f | \underset{\sim}{x}_f)$ $= B^T \underset{\sim}{x}_f = (\underset{\sim}{\beta}_1^T \underset{\sim}{x}_f, \ldots, \underset{\sim}{\beta}_m^T \underset{\sim}{x}_f)^T$ and covariance matrix $\text{Cov}(\underset{\sim}{z}) = \Sigma_{\underset{\sim}{\epsilon}}$. If we had iid $\underset{\sim}{z}_1, \ldots, \underset{\sim}{z}_n$, where $\underset{\sim}{z}_i = B^T \underset{\sim}{x}_f + \underset{\sim}{\epsilon}_i$, could use the prediction regions from §5.2. Typically have 0 cases with $\underset{\sim}{x}_i = \underset{\sim}{x}_f$. Instead, use $\hat{\underset{\sim}{z}}_i = \hat{B}^T \underset{\sim}{x}_f + \hat{\underset{\sim}{\epsilon}}_i$ $= \hat{\underset{\sim}{y}}_f + \hat{\underset{\sim}{\epsilon}}_i$, for $i = 1, \ldots, n$. This takes the data cloud of n residual vectors $\hat{\underset{\sim}{\epsilon}}_i$ and centers the cloud at $\hat{\underset{\sim}{y}}_f$. The nonparametric region based on the $\hat{\underset{\sim}{z}}_i$ can be shown to be a large sample $100(1-\delta)\%$ prediction region for $\underset{\sim}{y}_f$ given $\underset{\sim}{x}_f$.

want $n \geq \max[K(m+p)^2, mp+30, 10p]$ eg with $K=3$

ex) $Y_1 = \log S$, $Y_2 = \log M$,

$X_2 = L$, $X_3 = \log W$, $X_4 = H$

as in HW11 B).

$$\hat{B} = \begin{bmatrix} -2.3224 & -2.7365 \\ 0.0048 & 0.0024 \\ 1.1254 & 0.8504 \\ 0.0137 & 0.0162 \end{bmatrix}$$

Predict $\underset{\sim}{Y_f}$ if $\underline{X}_f = (1, 250, 4, 100)^T$

$$\hat{Y}_1 = \underline{\hat{B}_1}^T \underline{X}_f = -2.3224(1) + 0.0048(250) + 1.1254(4)$$

$$+ 0.0137(100) = 4.749$$

$$\hat{Y}_2 = \underset{\sim}{\hat{\beta}_2}^T \underset{\sim}{X}_f = -2.7365(1) + 0.0024(250) + 0.8504(4)$$

$$+ (.0162)100 = 2.885.$$

so $\underset{\sim}{\hat{Y}_f} = \begin{pmatrix} 4.749 \\ 2.885 \end{pmatrix}$

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18) $\underset{\sim}{\hat{\Sigma}_\varepsilon} = \frac{1}{n-p} \hat{E}^T \hat{E}$, $\quad S_r = \frac{n-p}{n-1} \underset{\sim}{\hat{\Sigma}_\varepsilon} = \frac{\hat{E}^T \hat{E}}{n-1}$

= sample covariance matrix of the residuals $\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_m$.

S_r is a $\sqrt{n}$ consistent estimator of Σ_ε.