Math 583 HW 4 Fall 2017. Due Wednesday, Sept. 27.

Exam 1 is on Friday, Sept. 22. Quiz 4 on Friday, Sept. 29 is similar to HW 4. Use 3 sheets of notes.

Problem numbers are from the Olive text.

A) 3.8. For the output below, an asterisk means the variable is in the model. All models have a constant, so model 1 contains a constant and mmen.

a) List the variables, including a constant, that models 2, 3, and 4 contain.

b) The term out$cp lists the C_p criterion. Which model (1, 2, 3, or 4) is the minimum C_p model I_{min}?

c) Suppose $\hat{\boldsymbol{\beta}}_{I_{min}} = (241.5445, 1.001)^T$. What is $\hat{\boldsymbol{\beta}}_{I_{min},0}$?

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Selection Algorithm: forward #output for Problem 3.9
         pop mmen mmilmen milwmn
1  ( 1 ) " " "*"  " "     " "
2  ( 1 ) " " "*"  "*"     " "
3  ( 1 ) "*" "*"  "*"     " "
4  ( 1 ) "*" "*"  "*"     "*"
out$cp
[1] -0.8268967  1.0151462  3.0029429  5.0000000
```

B) 3.9. Consider the output above Olive (2017) Example 2.7 for the OLS full model. The column *resboot* gives the large sample 95% CI for β_i using the shorth applied to the $\hat{\beta}^*_{ij}$ for $j = 1, ..., B$ using the residual bootstrap. The standard large sample 95% CI for β_i is $\hat{\beta}_i \pm 1.96 SE(\hat{\beta}_i)$. Hence for β_2 corresponding to L, the standard large sample 95% CI is $-0.001 \pm 1.96(0.002) = -0.001 \pm 0.00392 = [-0.00492, 0.00292]$ while the shorth 95% CI is $[-0.005, 0.004]$.

a) Compute the standard 95% CIs for β_i corresponding to log(W), H, and log(S). Also write down the shorth 95% CI. Are the standard and shorth 95% CIs fairly close?

b) Consider testing $H_0 : \beta_i = 0$ versus $H_A : \beta_i \neq 0$. If the corresponding 95% CI for β_i does not contain 0, then reject H_0 and conclude that the predictor variable X_i is needed in the MLR model. If 0 is in the CI then fail to reject H_0 and conclude that the predictor variable X_i is not needed in the MLR model given that the other predictors are in the MLR model.

Which variables, if any, are needed in the MLR model? Use the standard CI if the shorth CI gives a different result. The nontrivial predictor variables are L, log(W), H, and log(S).

C) 3.3. For ridge regression, let $\boldsymbol{A}_n = (\boldsymbol{W}^T\boldsymbol{W} + \lambda_{1,n}\boldsymbol{I}_{p-1})^{-1}\boldsymbol{W}^T\boldsymbol{W}$ and $\boldsymbol{B}_n = [\boldsymbol{I}_{p-1} - \lambda_{1,n}(\boldsymbol{W}^T\boldsymbol{W} + \lambda_{1,n}\boldsymbol{I}_{p-1})^{-1}]$. Show $\boldsymbol{A}_n - \boldsymbol{B}_n = \boldsymbol{0}$.

Problem D) is on the back.

D) 3.18. The *R* program generates data satisfying the MLR model

$$Y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + e$$

where $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)^T = (1, 1, 0, 0)$.

a) Copy and paste the commands for this part into *R*. The output gives $\hat{\boldsymbol{\beta}}_{OLS}$ for the OLS full model. Give $\hat{\boldsymbol{\beta}}_{OLS}$. Is $\hat{\boldsymbol{\beta}}_{OLS}$ close to $\boldsymbol{\beta} = 1, 1, 0, 0)^T$?

b) The commands for this part bootstrap the OLS full model using the residual bootstrap. Copy and paste the output into *Word*. The output shows $T_j^* = \hat{\boldsymbol{\beta}}_j^*$ for $j = 1, ..., 5$.

c) $B = 1000$ T_j^* were generated. The commands for this part compute the sample mean $\overline{T}^*$ of the T_j^*. Copy and paste the output into *Word*. Is $\overline{T}^*$ close to $\hat{\boldsymbol{\beta}}_{OLS}$ found in a)?

d) The commands for this part bootstrap the forward selection using the residual bootstrap. Copy and paste the output into *Word*. The output shows $T_j^* = \hat{\boldsymbol{\beta}}_{I_{min},0,j}^*$ for $j = 1, ..., 5$. The last two variables may have a few 0s.

e) $B = 1000$ T_j^* were generated. The commands for this part compute the sample mean $\overline{T}^*$ of the T_j^* where T_j^* is as in d). Copy and paste the output into *Word*. Is $\overline{T}^*$ close to $\boldsymbol{\beta} = (1, 1, 0, 0)$? It is likely that the last two entries of $\overline{T}^*$ are closer to 0 than the last two entries in c).