

# Math 104: Finite Mathematics

## Summer 2002, Final Exam

This exam is meant to test your mastery of stuff we've done in class. If at any point it looks like I'm trying to trick you, or if it looks like you'd need something we haven't done, that's probably a clue that you're looking at it the wrong way. Of course, you're welcome to ask me if you have any questions about the statements of the problems.

Answer each question, *showing all work which is necessary to do the problem* (i.e. "my calculator says so" does not suffice). Answers may be left in the form of factorials or as products, sums, square roots, etc. of numbers unless otherwise noted.

### 1 New material since test 2

- (20 pts) Corey is on his way to see his sweetheart when he suddenly thinks, "Egads! Is it her birthday?" He really can't remember. He figures he has time to pick up flowers for her if he needs to. If it is not her birthday and he doesn't bring flowers, there's no harm (he gives it a value of zero). If it is not her birthday and he does bring flowers, she'll be pleased, so he gives that a value of +1. If it is her birthday and he brings her flowers, he will have demonstrated his caring and sensitivity, and he gives that a value of +1.5. If it is her birthday and he shows up emptyhanded, he's in deep ... trouble (-10). As he is considering his options, he meets you on the sidewalk. "Quick!" he says, "What should I do?"
  - What is the payoff matrix for this game? [Hint: Nature may choose to make it her birthday or not.]
  - Is the game strictly determined? If so, give the saddle point.
  - What is Corey's best pure strategy?
- (15 pts) In her new job as head bigshot of Notre Dame computing, Emily learns that the network has received a strange message from Ross's computer. She knows that Ross meant to send the message  $x$ , either  $[1 \ 0 \ 0]$  or  $[1 \ 1 \ 1]$ . In order to allow errors in transmission to be detected, Ross's computer would have multiplied the message by

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

and then sent  $xG$ . Emily received  $[1 \ 1 \ 0 \ 2 \ 2]$ , and assumes that the message must have been garbled in transmission. She's called to ask for your help.

- (a) What are the two possibilities for what Ross sent (i.e. for  $xG$ )?
- (b) Which is more similar to the message Emily received (and thus more likely to have been what Ross really sent)?
3. (20 pts) Nick and Brian call you on the telephone one evening. They are starting a used textbook business, and they have warehouses in Nashville and Los Angeles. The cost per box to ship books is as follows:

Nashville to Walla-Walla	\$12
Nashville to Ft. Hood	7
Los Angeles to Walla-Walla	8
Los Angeles to Ft. Hood	10

They have fifty boxes of *Finite Mathematics and its Applications* in Nashville, and seventy in LA. They have orders for eighty-five boxes in Walla-Walla and twenty-five in Ft. Hood. What is the cheapest way to supply the needed books to the two locations? Along the way, be sure to tell us

- (a) What is the feasible set?
- (b) What is the objective function?
- (c) What is the cost of shipping according to your solution?
4. (20 pts) Allison and Joey have decided to make some extra money this summer by working as analysts for the Air Force. Colonel John Reed, the director of their project, says, “Here’s how it is: Our bombers fly in pairs. One will fly in the lead position, the other will fly in the wing position. One carries the bomb, and the other carries jamming equipment. Our target has the capability to shoot down one bomber, but not both. We estimate the following payoff matrix:”

		Shot down	
		lead plane	wing plane
Bomb in	lead plane	0	3
	wing plane	3	0

“By the way, some of the folks in the back room thought you might want to know that the lines  $0y_1 + 3y_2 = 1$  and  $3y_1 + 0y_2 = 1$  intersect at the point  $(\frac{1}{3}, \frac{1}{3})$ . I can’t imagine why. It’s all greek to me. What I want to know is,

- (a) What is the expected value if the target shoots down the lead plane with probability .75 and the bomb is in the lead plane with probability .25?
- (b) How should we choose where to put the bomb?”

## 2 Older material

5. (10 pts) Consider the universe

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and the following sets:

$$R = \{\text{odd members of } U\}$$

$$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$T = \{0, 2, 3, 5, 7, 8\}$$

- (a) Draw a venn diagram to represent this situation, writing each member of  $U$  in the appropriate region.
- (b) List the elements in the following sets:
- $R \cap S$
  - $R \cup S$
- (c) Use  $R, S, T$ , and set-theoretic notations to describe the set  $\{2, 8\}$ .
6. (10 pts) Consider the data set

$$4, 0, 0, 3, 7, 3, 5, 1, 1, 7, 3, 3, 1, 3, 9, 4, 8$$

Give the five-number summary ( $Q_0, Q_1, Q_2, Q_3$ , and  $Q_4$ ) of this data set, and draw a corresponding box plot.

7. (8 pts) When researching affirmative action, I found 846 law review articles dealing with affirmative action in either education or employment. Six hundred twelve of them talked about affirmative action in employment. Seven hundred forty five talked about affirmative action in education. How many talked about both?
8. (10 pts) Suppose my CD case has 6 CD's. Two of them are by U2, and four are by other artists. Suppose that you pick 2 CD's at random and observe how many of the ones picked are by U2.
- Write the probability distribution for the experiment.
  - Find the mean of this distribution.
  - Find the population standard deviation ( $\sigma$ ) of this distribution.
  - What is the probability that you get either 1 or 2 CD's by U2?
9. (7 pts) What is the value of  $\frac{7!5!}{4!5!}$ ? [A number]
10. (10 pts) Rick and Katy have rounded up some friends for baseball. Rick's team has 15 players. How many batting orders (of 9 players) are possible for them?

11. (10 pts) Kathryn and Zhu Jun have become aware of a cruel psychology experiment being conducted on campus. In this experiment, a student is asked two questions: First, "Guess a number from 1 to 3 inclusive." Second, "Is the glass half empty or half full?" The student may or may not receive an electrical shock. Observe the answers, and observe whether or not the student received an electrical shock.
- (a) Write the elements of the sample space.
  - (b) Circle the event that either the student guessed 2 or the student received an electrical shock.
  - (c) Calculate the probability of the event that either the student guessed 2 or the student received an electrical shock.
12. (10 pts) The probability of  $E$  is 0.75. The probability of  $F$  is 0.5. The probability of  $E \cap F$  is 0.4. Are  $E$  and  $F$  independent? What is  $Pr(E|F)$ ?